

**ICCAE**

**Military Technical College
Kobry Elkobbah,
Cairo, Egypt**

**7th International Conference
On Civil & Architecture
Engineering**

ASSESSMENT OF SLOPE STABILITY USING FEM

Tarek N. Salem¹, Khaled Hassan², Mohsen Mashhour³, Ahmed AbuEiella⁴

ABSTRACT

Slope stability is one of the fundamental problems faced on a consistent basis by the majority of practicing geotechnical engineers. The studied slope may be a cut slope (excavation), or a formed slope (embankment). The objective of the current paper is to evaluate and assess the effect of many parameters affecting the stability of slopes and to produce a general equation for calculating the safety factors in simple and easy way. The finite element method is a very effective and versatile numerical tool, especially in nonlinear analysis, and consequently, a finite element code for soil and rock analysis (PLAXIS) is chosen for this paper. Results indicated that the slope height is probably the most effective parameter in the safety factors of slopes, followed by the effect of the presence of water beside or within the slope. Shear strength parameters and slope angle also affect the safety factor value, as indicated in the developed general equation for calculating the factor of safety in homogeneous soil slopes. The proposed equation is verified by comparing the calculated values obtained from that equation with PLAXIS results, and the maximum difference was about $\pm 4\%$. Moreover, the results obtained from the proposed equation are compared with the results from Michalowski Charts (2002) and the maximum difference was about 7% in the conservative side, despite employing different analysis approaches.

KEY WORDS

^{1,2} Assoc. Prof., Dep. of Structural Eng., Faculty of Eng., Zagazig University, Zagazig, Egypt.

³ Prof. Dep. of Structural Eng., Faculty of Eng., Zagazig University, Zagazig, Egypt.

⁴ Ass. Lecturer, Dep. of Structural Eng., Faculty of Eng., Zagazig University, Zagazig, Egypt.

Slope Stability; Numerical Analysis; Regression Analysis, Generalized Equation.

1. INTRODUCTION

A quantitative and qualitative assessment of the stability of a slope is clearly important when a verdict is needed about whether the slope is stable or not, and discussions are to be made as a consequence, Nash [1]. Salem and Al-Tuhami [2], presented nonlinear FEM analysis of slopes in which the failure surface was not assigned in advance. The finite element method is used, employing Mohr-Coulomb elasto-plastic model with a non-associated flow rule. Results obtained using the FEM solution of slopes were very close to those obtained using simplified methods like Bishop and Janbu, indicating the good accuracy of the simplified techniques.

Lane and Griffiths [3], presented an assessment of stability of slopes under drawdown conditions by using finite element program FE-EMB1LG. Al-Tuhami and Salem [4], presented a simple equation based on the finite element results, only as a first trial to present an equation for obtaining the factor of safety in slopes. The authors used a huge data of slope characteristics, geometry and configuration to assess the factor of safety for a proposed slope.

2. PARAMETRIC STUDY

In this study, a loaded embankment of homogenous single soil layer is modeled. The slope is mainly c - ϕ soil slope with soils possessing both cohesion and internal friction. Higher cohesion values and lower friction angles indicate short term analysis, while lower cohesion and higher friction angles indicate long term analysis. For the sake of completeness and to cover most of the practical soil parameters range and slope configuration, huge number of analyses has been performed. Fig. 1 illustrates a schematic diagram for the studied slope geometry and configuration.

Where:

H = Slope height, varying from 5 to 20 meters;

H_w = Height of water in front of the slope, varying from 0 to H ;

β = Slope angle, varying from 18 to 60 degrees; and

n = Factor expressing the loaded length slope as a percentage of the slope height, with values of 0, 0.1, 0.2, up to 1.8.

The soil properties are chosen to be in the practical range that simulates an actual geotechnical analysis, and these properties are:

c = Soil cohesion ranging between 15 and 100 (kN/m^2) to cover soil consistencies ranging from soft to stiff;

ϕ = Soil angle of internal friction ranges between 10 to 40° to cover the soil relative densities from very loose to dense soils; and

ψ = Angle of soil dilation (degrees), taken to be $\phi - 30^\circ$, for ϕ less than 30°, the angle ψ is taken equal to zero.

The following parameters are chosen according to the chosen soil type (consistency and/or relative density), with values determined from well defined correlations:

E = Soil stress-strain modulus, (kN/m^2); G_s = Soil shear modulus (kN/m^2);

γ_d = Dry unit weight (kN/m^3); γ_{sat} = Saturated unit weight (kN/m^3);
 ν = Poisson's ratio; and k = Coefficient of permeability (m/day).

The resulting number of performed program runs was about fifteen thousand runs to cover almost all the above mentioned combinations of soil parameters.

3. ANALYSIS AND RESULTS

The previously mentioned parameters are input into the finite element code (PLAXIS) [5]. Huge data volumes are analyzed using regression analysis to produce a general equation as a simple form used in obtaining the safety factors, AbuEiella [6]. The effect of each parameter on the factor of safety is presented separately hereinafter.

3.1 Effect of Shear Strength Parameters

No doubt, as a soil mechanics fact, that increasing the soil shear strength parameters (c , ϕ) has a great influence in increasing the safety factor of slopes, because increasing these parameters results in an increase in the soil shear strength (ϕ), consequently, increasing the slope resistance to failure.

3.1.1 Effect of soil cohesion (c)

In short term (undrained analysis), the soil cohesion values are high with no or small values of angle of friction (ϕ) and vice versa for drained or long term analysis. Keeping all other parameters constant and increasing the soil cohesion only resulted in an increase in the safety factor (F.S) of slopes, Fig. 2 indicates this fact. The relation between (F.S) and the soil cohesion (c) is approximately linear and the soil cohesion is directly proportional to the slope safety factor. If the soil cohesion is increased by 100% (representing stiffer soils), the average percentage of increase in the safety factor ranges between 35% and 55%. However, for smaller values of angle of internal friction, the percentage of increase in the safety factor is larger than that at higher values of the angle of internal friction.

3.1.2 Effect of soil angle of internal friction (ϕ)

Considering the soil shear strength as simply two fold parameters, increasing any of them will enhance the soil slope performance against stability failure. In addition to soil cohesion, the other shear strength parameter is the soil angle of internal friction (ϕ). Increasing (ϕ) will result in a consequent increase in the safety factor of soil slopes, giving the same trend of increasing the soil cohesion (c).

Fig. 3 simply indicates that, when the soil angle of internal friction increased, the safety factors increased. If the soil angle of internal friction (ϕ) increased by 100% an average increase of about 30 to 50% in the factor of safety is noticed. This observation indicates a direct proportion of the factor of safety with the soil angle of internal friction (ϕ). The relation between the safety factor and the soil angle of internal friction (ϕ) is approximately linear, as that of soil cohesion (c).

3.2 Effect of Slope Height

Fig. 4 presents the relation between the slope height (H) and the slope factor of safety (F.S). The relation between the safety factor and the slope height is an inversely proportional relation in which increasing the slope height results in decreasing the safety factor. For relatively smaller slope heights (5-10) meters, a pronounced increase in the safety factor is obtained with increasing the soil shear strength, but for larger slope heights, a smaller increase is obtained. Increasing the slope height resulted in an increase in the disturbing force, and consequently the factor of safety decreases. For larger slope heights, the soil failure wedge is relatively small, and the failure surface is shallow toe failure and/or face failure. The failure surface area for this type of failure is relatively small, and a relatively small disturbing force will induce slope failure. This may propose an interpretation of the small gap between curves representing large slope heights, and the wide gap between curves representing smaller heights. Fig. 5 indicates the same results.

3.3 Effect of Slope Angle (β)

The effect of slope angle on the safety factor is presented in the forthcoming Fig.s. Fig. 6 depicts the relation between the soil angle of internal friction (ϕ), and the (F.S) for different slope angles. Fig. 7 represents the relation between slope angle and (F.S). The relation between the slope angle (β) and (F.S) is an inversely proportional relation, such that increasing the slope angle results in a decrease in the (F.S). Consequently, gentle slopes are more stable than the steeper ones because the disturbing force is increased due to increasing the slope angle. For gentle slopes, increasing the shear strength parameters has a more pronounced effect in the safety factor than for steeper slopes.

3.4 Effect of the Presence of Loads on Slope

In addition to the own weight (gravity loads) of the slope surface loads are added on the embankment. Fig. 8 illustrates the effect of surface loads on the stability of slopes. The imposed load is encountered in many loading conditions such as an equivalent static load from traffic and buildings on slopes. Increasing the loaded length, which expresses the area covered by the load starting from the slope crest, resulted in decreasing the safety factors until reaching the end of the failure wedge. Beyond the failure wedge, there is no effect of the loaded length on the safety factors of slopes. It should be noted that the failure surface and its extension is affected, to some extent, by the presence of that surface load and its value. The factor (n) expresses the load extension from the slope crest ranging from zero to 1.9 H, where H is the slope height. The load intensity affects the safety factor, such that if the load

value is increased, the (F.S) will decrease due to increasing the disturbing force, which affects the overall stability of the slope.

Fig. 9 shows that the relation between the load intensity and the factor of safety (F.S) is a nonlinear one, if the load value is increased by 10 folds, which is an exaggerated case, the (F.S) will be decreased by nearly 20% only. The surface loading value is well known to be a disturbing force, which consequently decreases the safety factor (F.S). Therefore, the relation between the values of surface static loads and the loaded length from the slope crest, which is expressed as the loading area, are inversely proportional with (F.S).

3.5 Effect of Water Level in the Waterway

Fig. 10 illustrates the effect of water level beside the slope on its stability. It is clear that higher water levels above toe resulted in higher values of safety factors. On the other hand, the most critical case for the safety factor is encountered when the waterway is empty and the embankment is still fully saturated, which is known as the rapid drawdown condition. The figure indicates that the factor of safety increased with increasing the factor (k), which represents the water height to the slope height. In addition, the lowest values of (F.S) are obtained when the factor ($k = 0$), indicating the most critical case, and the highest safety values are obtained when the factor ($k = 1$). It is also noticed that the gap between the curves increased with increasing the factor (k). Under the initial conditions, total saturation is assumed at $k = 1.0$. The water in the waterway induces a hydrostatic pressure at the face of slope that results in an increase in the stabilizing force. If a rapid water drawdown took place while the slope remains saturated, at least for a while, this condition reduces the hydrostatic pressure at the slope face. Thus, removing the water in front of the slope face reduces the hydrostatic pressure proportional to the amount of drawdown. Then the total resisting stress on the failure surface decreased, thereby necessitating the mobilization of some additional shear resistance along the failure surface in order to compensate the decreasing resisting force. If this needed shear resistance is not available, the stability of the slope may be spoiled. Therefore, the critical case took place when the waterway is completely empty and the slope is fully saturated (a typical rapid drawdown case).

3.6 Effect of (E, Soil Stress-Strain Modulus and ψ Dilation Angle)

Theoretically, it could be argued that stiffness parameters should be irrelevant for the factor of safety in a FEM analysis. It is confirmed, through huge number of runs, that varying the (E) modulus in PLAXIS does not affect the safety factor value, as long as one homogeneous soil layer is assumed. This may not be true for layered slopes. The effect of the stress-strain modulus only has a great effect, as a strain modulus, on displacements and strains. The Mohr-Coulomb material model in PLAXIS requires the value of the dilation angle as an input parameter. Many of the available limit load solutions in plasticity theory assume an associated plastic flow rule, which means that the dilation angle equal to the angle of internal friction (ϕ). Laboratory tests demonstrate that the angle of dilation is much smaller than the angle of internal

friction and often $\psi = \text{zero}$ is used (Manzari and Nour [7]). However, in the employed soil model the dilation angle equals zero for values of $\phi \leq 30^\circ$ and equal to $(\phi - 30)$ when $\phi > 30^\circ$. Although this rule is suggested in the PLAXIS manual, one can input any value for the angle of dilation as an input data. Moreover, using $\psi = \text{zero}$ and $\psi = \phi$ resulted in only 3% increase in the factor of safety. Based upon a number of performed runs using the software PLAXIS to explore the effect of (E and ψ) on the safety factors in slopes, it is suggested to marginalize their effect due to their less pronounced effect.

4. REGRESSION ANALYSIS

The discussion in this part deals mainly with the use of regression analysis, as a means to present a simple equation for determining the factor of safety in homogeneous soil slopes, based on the huge database of analyzed slopes using (F.E.M) through PLAXIS program.

Regression analysis is used to establish a general equation between different parameters, i.e., to quantify the dependent parameter based on the independent parameters values. However, to obtain the proposed equation these parameters should be presented in dimensionless form. Looking profoundly in the discussed parameters and their effect on the safety factor, some parameters are directly proportional, while others are inversely proportional to the factor of safety. For example, the safety factor is inversely proportional to the slope height (H) so a dimensionless parameter can be put as $(c/\gamma H)$, where c is the soil cohesion and γ is the soil unit weight, knowing that the safety factor is directly proportional to the shear strength parameters. In addition, the safety factor is directly proportional to the soil angle of internal friction (ϕ) and inversely proportional to the slope angle (β), so a dimensionless parameter can be presented to be $(\tan \phi / \tan \beta)$.

The value of surface load and its extension, which refers to the loading area, are inversely proportional to the factor of safety. So, two dimensionless parameters can be presented, these parameters are $(w/\gamma H)$ and (L_w/H) where (w) is the load value and (L_w) is the loaded length from the slope crest. The presence of water has a strategic influence on the safety factor so the water level in canal can be introduced in a dimensionless form as (H_w/H) , where (H_w) is the water height in the waterway.

A nonlinear regression analysis is performed using special purpose software called DATAFIT [8]. Results of fitting a huge number of data points showed good convergence with the proposed equation as presented with (R^2) value equals to 0.965. The factor, R^2 (coefficient of multiple determination) measures the proportion of variation in the data points, which is explained by the regression model. The data used in the regression analysis is 15190. The odd observations are only 290 records, these records are excluded to achieve higher value of (R^2) that these values have a diversion of about $\pm(15-20)\%$ from the calculated values using the proposed equation. It is worth mentioning that this diversion occurs for soils with exceptionally high shear strength parameters (c, ϕ). From the obtained results, a general equation for calculating the safety factors in slopes can be written in a simple form. To achieve this task, one can assume new abbreviations for the input parameters such as:

$$N_s = c/\gamma H, \quad F = \tan \phi / \tan \beta, \quad R_u = H_w/H, \quad G = w/\gamma H, \quad \text{and} \quad L = L_w/H$$

So the equation can be written as:

$$F.S = e^{((2.914*N_s) + (0.49*F) + (0.57*R_u) - (0.609*G) - (0.063*L) - 0.327)} \dots\dots\dots (1)$$

The equation seems to be very long, but the strategy of introducing most of the parameters, affecting the slope stability analysis as in an individual equation is regarded. The first term expresses the soil cohesion ratio as divided by the unit weight and slope height, the second expresses the friction ratio, the third represents the water height ratio, the fourth term expresses the load ratio, and the fifth term refers to the loaded length ratio. Note that all the equation terms are in dimensionless form and the last term in this equation is a constant. As an advantage of writing down the above equation in that form is allowing an easy estimate for the needed parameters to be considered. In other words, if the load on the slope is thought to be the most critical, simply set the safety factor equal to 1, which means that the exponent term of the logarithm of base (e) is equal to zero. Substituting with the other known parameters will result in obtaining the needed critical parameter that will lead to a just stable slope.

Michalowski [9] provided stability charts to establish the (F.S) using the limit analysis method (LAM). Fig. 11 shows the proposed equation results versus Michalowski stability charts. The Fig. portrays the relation between the stability number (N_s), which refers to the term of ($c/\gamma H$), and the safety factor (F.S).

The solid line expresses the proposed equation obtained by regression analysis, whereas the values obtained from Michalowski charts are presented by dash lines. One can note the difference between the two techniques, the proposed technique as a finite element technique (F.E.M), and the limit analysis technique based on the kinematics approach. The proposed equation gives more conservative values for the safety factors by about 7% less than the values obtained from the limit analysis approach, using Michalowski (2002) charts.

The obtained value of the safety factor (F.S) from the equation is the critical value and valid for the range of the mentioned values in the parametric study regarding the soil shear strength parameters, slope height, slope angle, height of water in the canal, and loading conditions. Fig. 12 illustrates the relation between the stability number (N_s) and the safety factor.

Dash lines represent the obtained values from PLAXIS and the solid line expresses the calculated values for the safety factor using the proposed equation (Equation 1). The calculated values differ from the obtained values by about ($\pm 4.5\%$) in some cases.

Fig. 13 shows the relation between the friction factor ($F = \tan \phi / \tan \beta$) and the corresponding safety factor (F.S) for the obtained and calculated values. It is clear that the difference is so small and the coefficient (R^2) reflects this fact, that $R^2 = 0.965$.

5. CONCLUSIONS

Based on the results obtained from finite element analysis using the computer code (PLAXIS), along the the performed regression analysis, the following conclusions can be drawn:

- 1- Soil Shear strength parameters (c, ϕ) are directly proportional to the safety factor. If the shear strength parameters increased by 100%, an average increase of about 35 to 50% in the safety factor will be obtained. The results are applicable for undrained analysis with higher cohesion and lower friction and drained analysis with lower cohesion and higher friction.
- 2- For relatively lower slope heights, H = 5 to 10 m, if the soil shear strength parameters are increased by 100%, an increase of the safety factor of about 100% would be obtained. Increasing the slope height from 5 to 10 meters, resulted in a reduction in the safety factor of about 35%, this reduction reached about 60% when increasing the slope height from 10 to 20 m. Therefore, slope height is an effective parameter in the slope stability problems and it is inversely proportional with the safety factor.
- 3- Smaller slope angles resulted in higher slope safety factors. For gentle slopes, the effect of increasing the soil shear strength parameters is more pronounced. However, for steeper slopes, increasing the shear strength parameters has slight effect on the safety factors.
- 4- The effect of the presence of surface loads on the safety factor is slight, that if the surface loads are increased by 10 times, a decrease in the safety factor of about 25% will be obtained. In the meantime, for very high loading values, the embankment bearing capacity should be checked along with the stability analysis. However, when the loads are applied at a range of about 1.5 H away from the slope crest, the loading extension or intensity has no apparent effect on the factor of safety of that slope.
- 5- The relation between the water level in the waterway and the safety factor is a nonlinear relation. The critical value of the safety factor is encountered when the slope is saturated and the waterway is empty (rapid drawdown case). On the other hand, for the highest water level, the increase in the factor of safety due to the presence of water in the water way is found to be about 100%.
- 6- Based on the finite element method (FEM), through the finite element code (PLAXIS), a general equation for the assessment of the factor of safety in slope stability problems is obtained. The proposed equation contains most of the parameters affecting the stability of slopes. This general equation is presented as:

$$F.S = e^{((2.914*Ns) + (0.49*F) + (0.57*Ru) - (0.609*G) - (0.063*L) - 0.327)} \dots\dots\dots (1)$$

6. REFERENCES

- [1] Nash, D., "A Comparative Review of Limit Equilibrium Methods of Stability Analysis", In: M.G. Anderson, and K.S. Richards, (Eds.), Slope stability, Wiley, New York, (1987).
- [2] Salem, T.N., and Al-Tuhami, A.A., "Finite Element Analysis of Slopes", Soil Mechanics and Foundations, J. of Egyptian Geotechnical Society, Vol. 9, No. 2, pp. 64-73, (1998).
- [3] Lane P.A., and Griffiths D.V., "Assessment of Stability of Slopes under Drawdown Conditions", ASCE, J. of Geotechnical and Geo-environmental Eng. Div., Vol. 126, No. 5, pp. 443-450, (2000).
- [4] Al-Tuhami, A.A., and Salem, T.N., "Stability of (c- ϕ) Slopes", Soil Mechanics and Foundations, J. of Egyptian Geotechnical Society, Vol. 12, No. 2, pp. 61-72, (2001)
- [5] PLAXIS, "Finite Element Code for Soil and Rock Analysis", Version 7.20, Plaxis B.V., (2001).
- [6] Manzari, M.T., and Nour, M.A., "On Significance of Soil Dilatancy in Slope Minimum Factor of Safety in Slope Stability Analysis", ASCE, J. of Geotechnical and Geo-environmental Eng. Div., Vol. 126, No. 1, pp. 75-80, (2000).
- [7] AbuEiella, A.A., "Numerical Assessment of Loaded and Reinforced Earth Slopes", M.Sc. Thesis, Faculty of Engineering, Zagazig University, Zagazig, Egypt, (2006)
- [8] DataFit, "Statistical Analysis Software", Ver. 8.20, Oakdale Engineering, Pennsylvania, USA, (2007).
- [9] Michalowski, R.L., "Stability Charts for Uniform Slopes", ASCE, J. of Geotechnical and Geo-environmental Eng. Div., Vol. 128, No. 4, pp. 351-355, (2002).

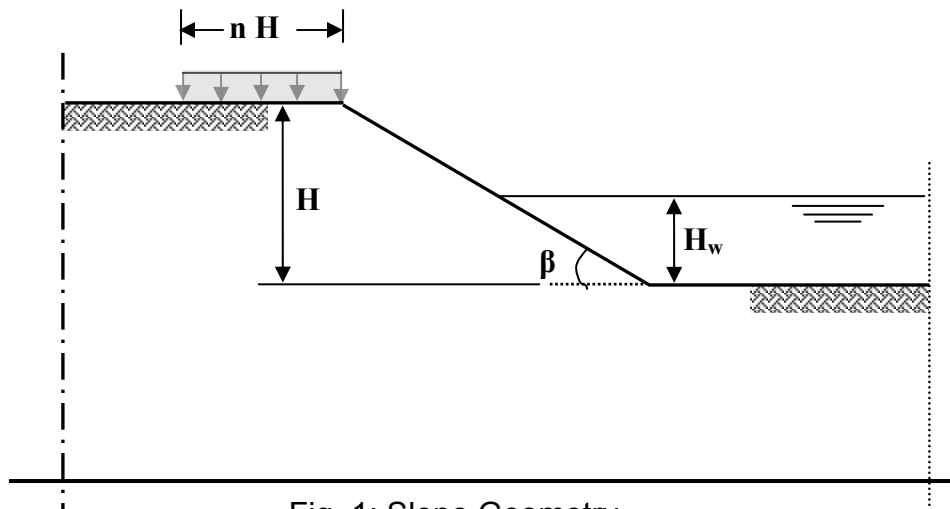


Fig. 1: Slope Geometry.

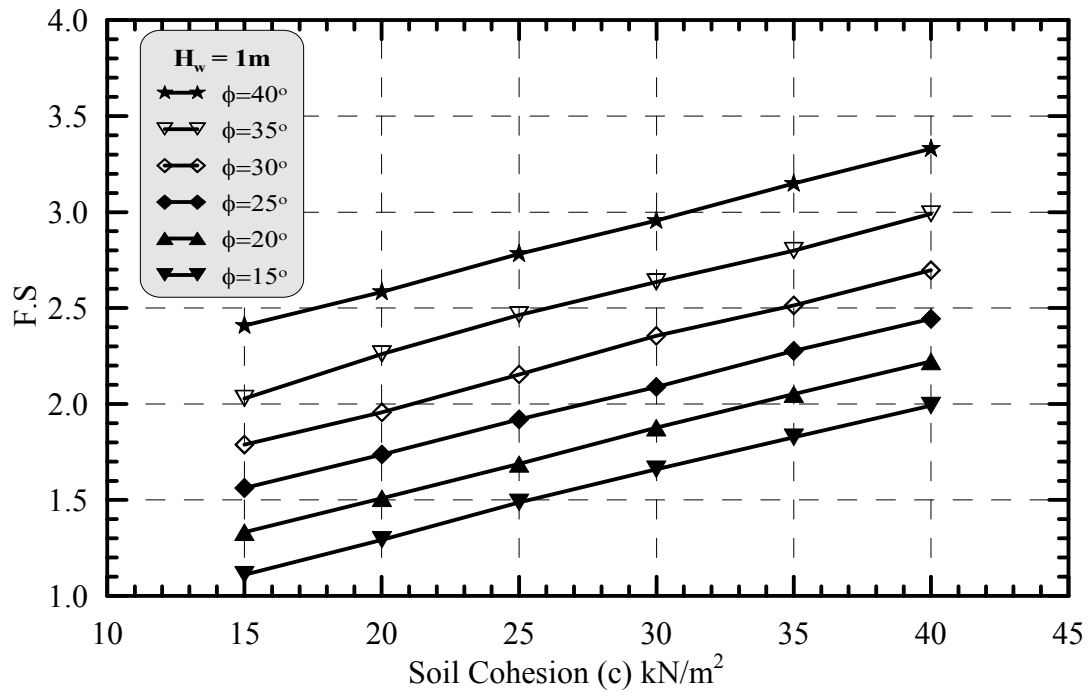


Fig. 2: Soil Cohesion versus F.S (H = 10m, Slope 2:1, H_w = 1m).

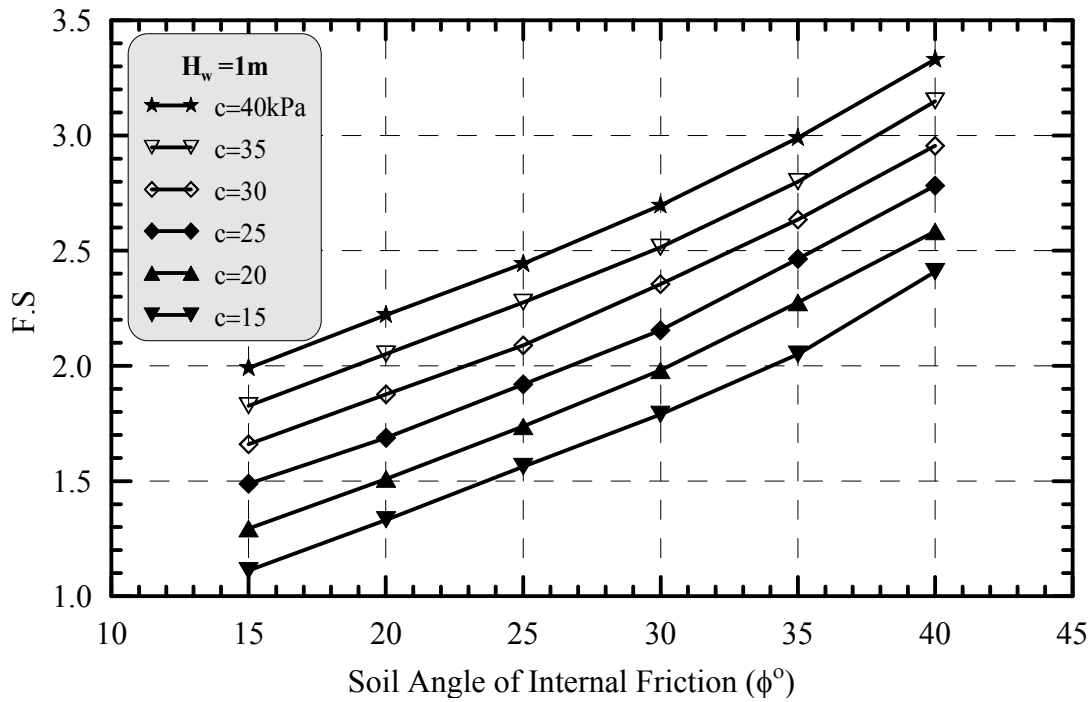


Fig. 3: Relation between (ϕ) and (F.S) (H = 10m, Slope 2:1, H_w = 1m).

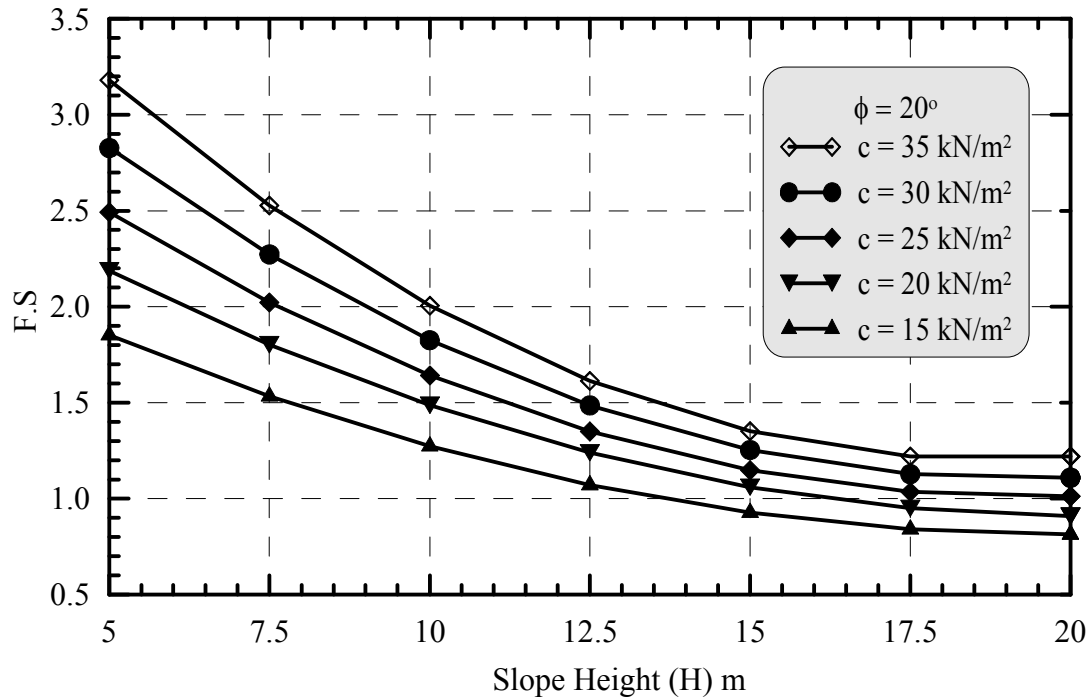


Fig. 4: Relation between Slope Height and (F.S) for Different Soil Cohesions, Slope 2:1 and $H_w = 0$.

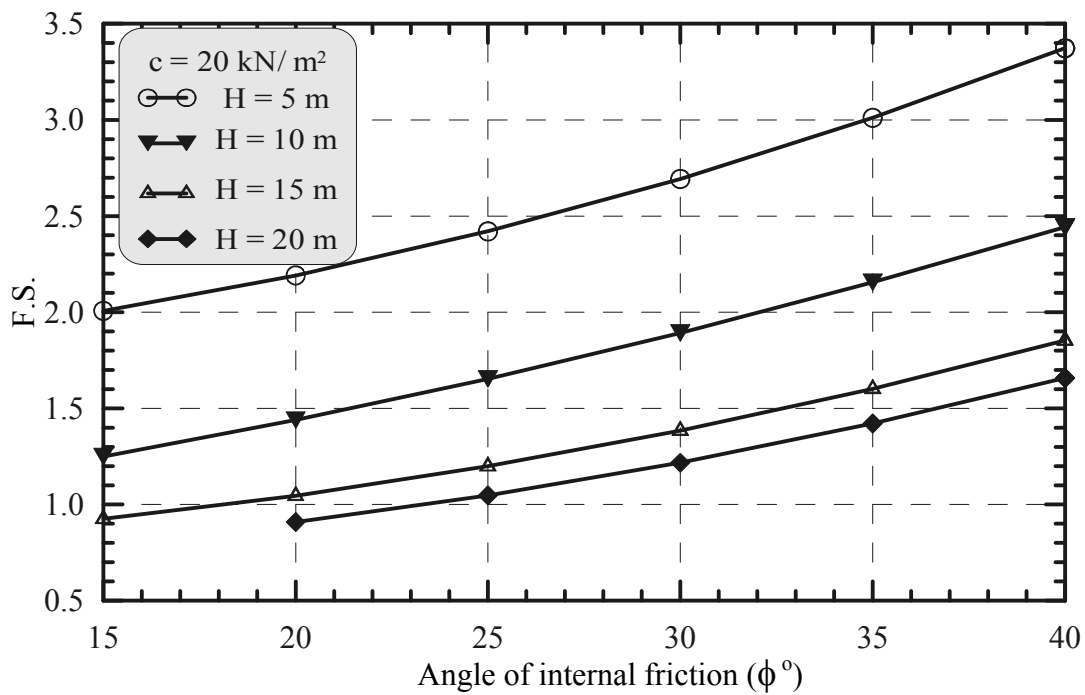


Fig. 5: Relation between (ϕ) and (F.S) for Different Slope Heights, Slope 2:1, $H_w = 0$.

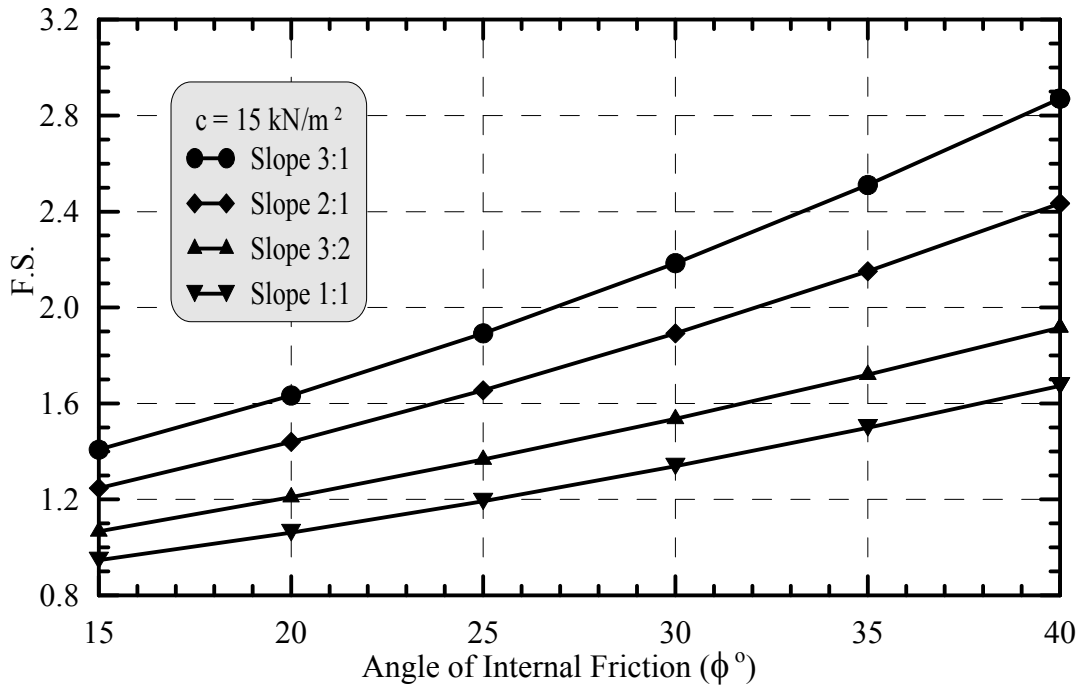


Fig. 6: Relation between (ϕ) and (F.S) for $c = 15 \text{ kN/m}^2$, $H = 10\text{m}$, $H_w = 10\text{m}$.

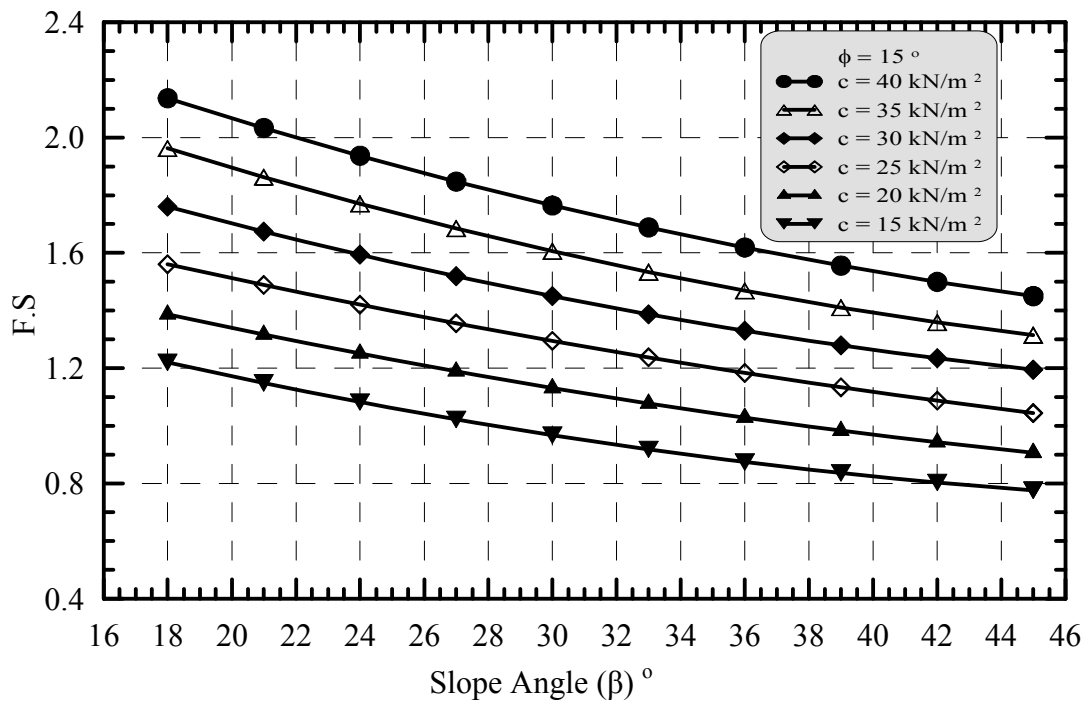


Fig. 7: Relation between Slope Angle (β) and (F.S) for Different Soil Cohesions, $H = 10 \text{ m}$, $H_w = 0$.

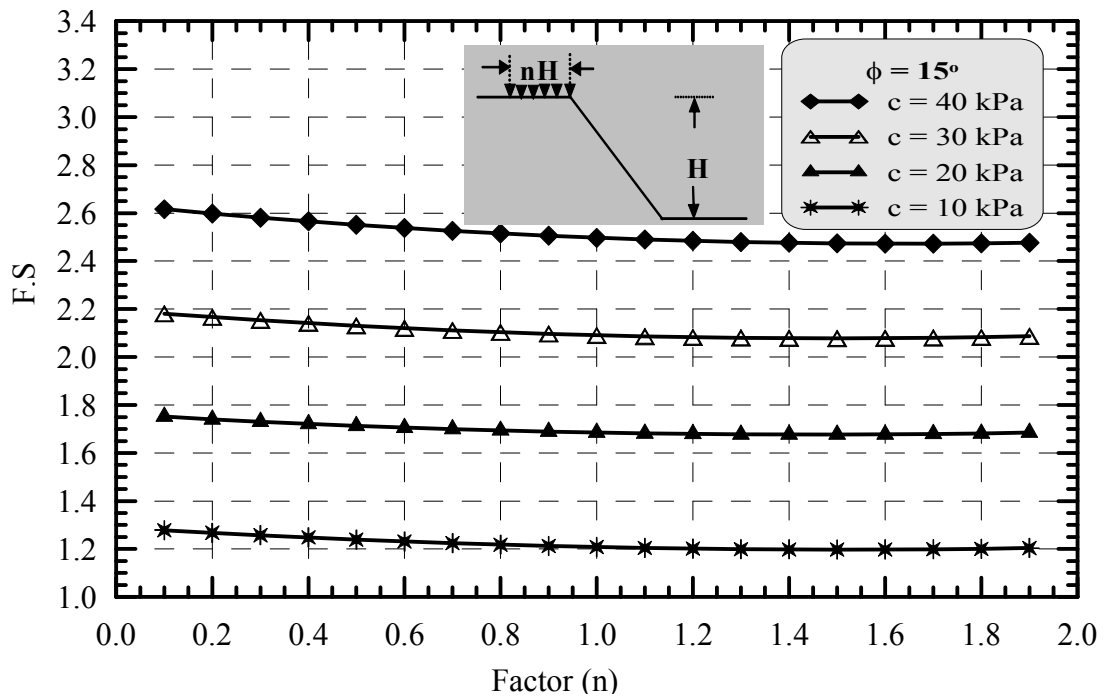


Fig. 8: Relation between the Load Extension Factor (n) and (F.S), Slope 3:2, H = 15 m.

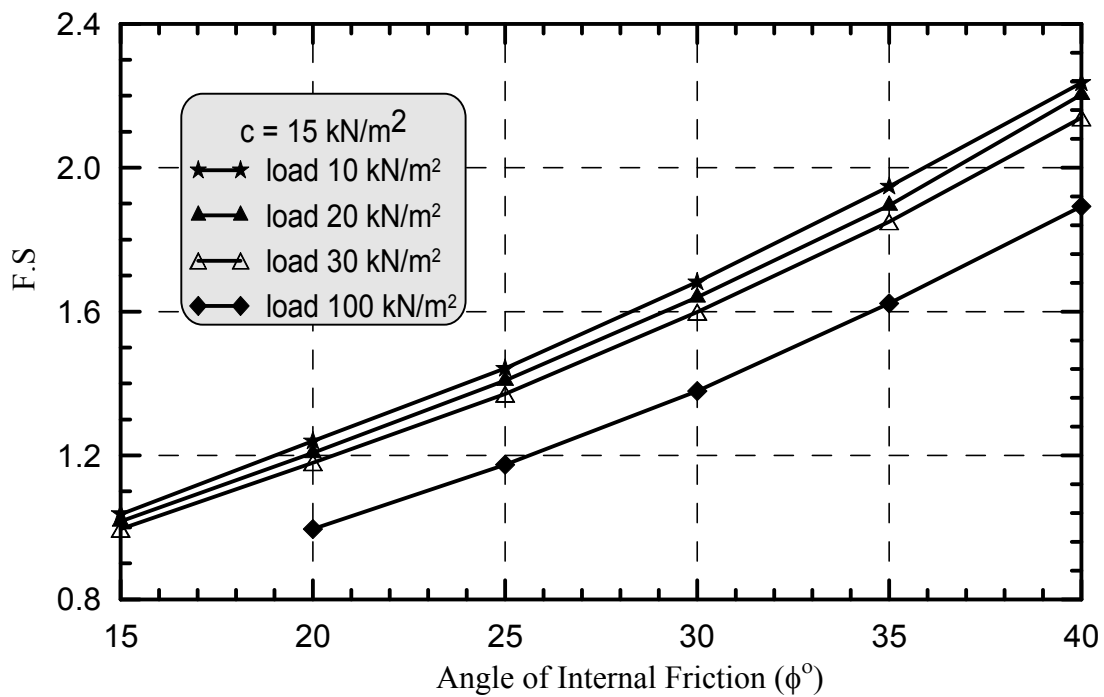


Fig. 9: Relation between (ϕ) and (F.S) for Loaded Slope for $c = 15$ kN/m², Slope 2:1, H = 10m, $H_w = 5$ m.

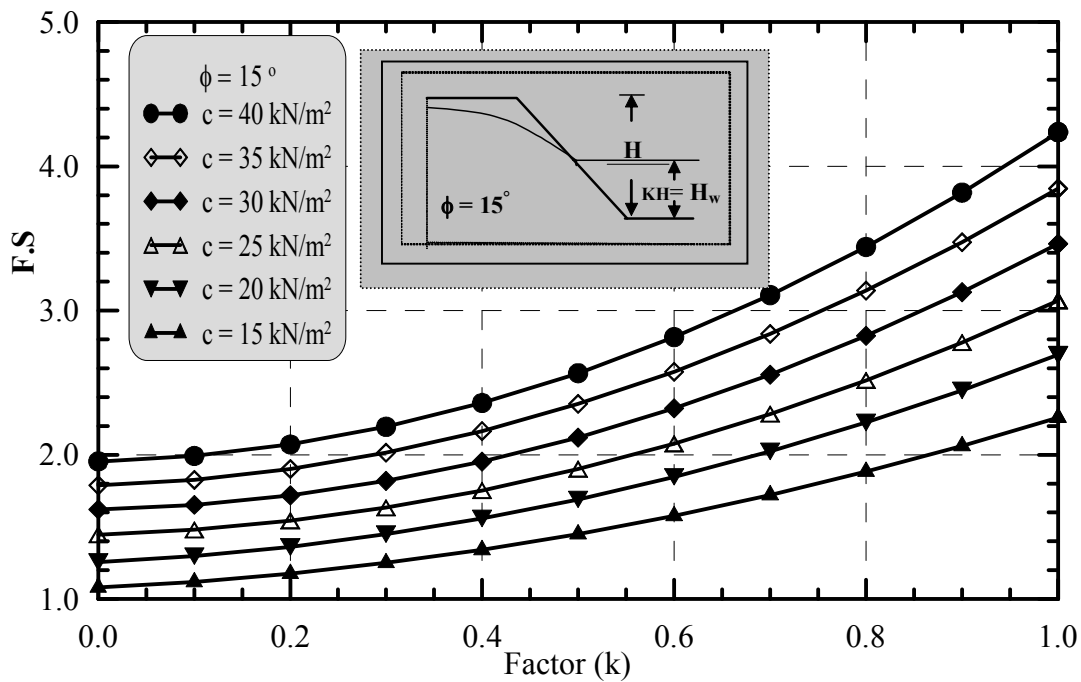


Fig. 10: Relation between (k) and (F.S) for Different c Values and $\phi = 15^\circ$.

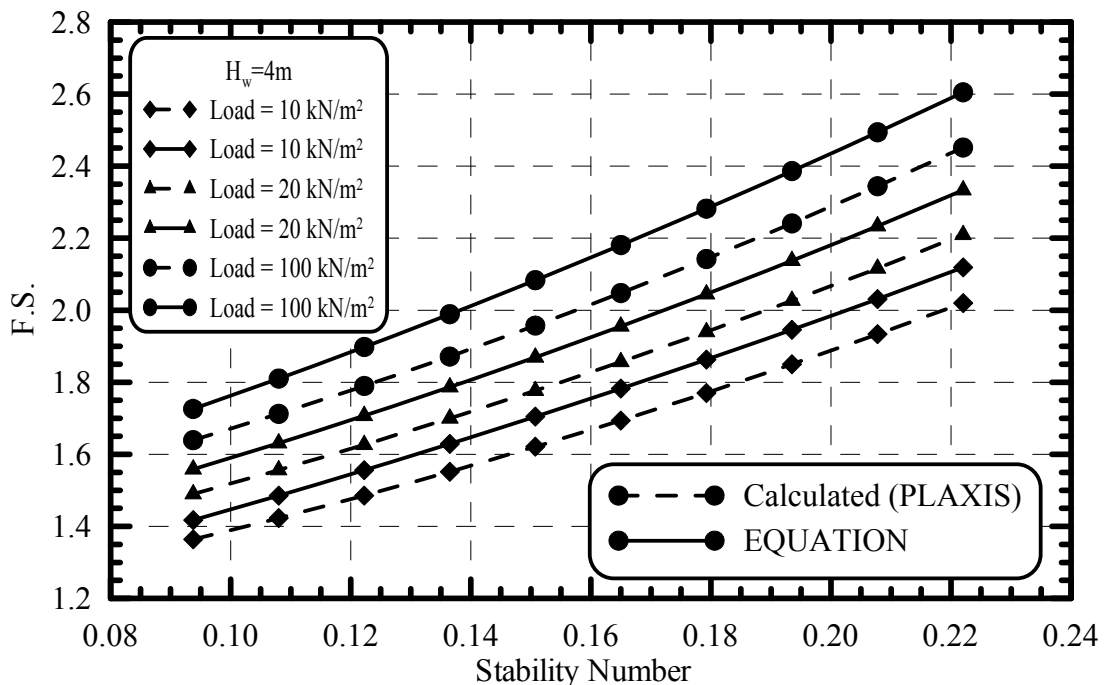


Fig. 11: Relation between (N_s) and (F.S) for the Proposed Equation and Michalowski Charts (2002).

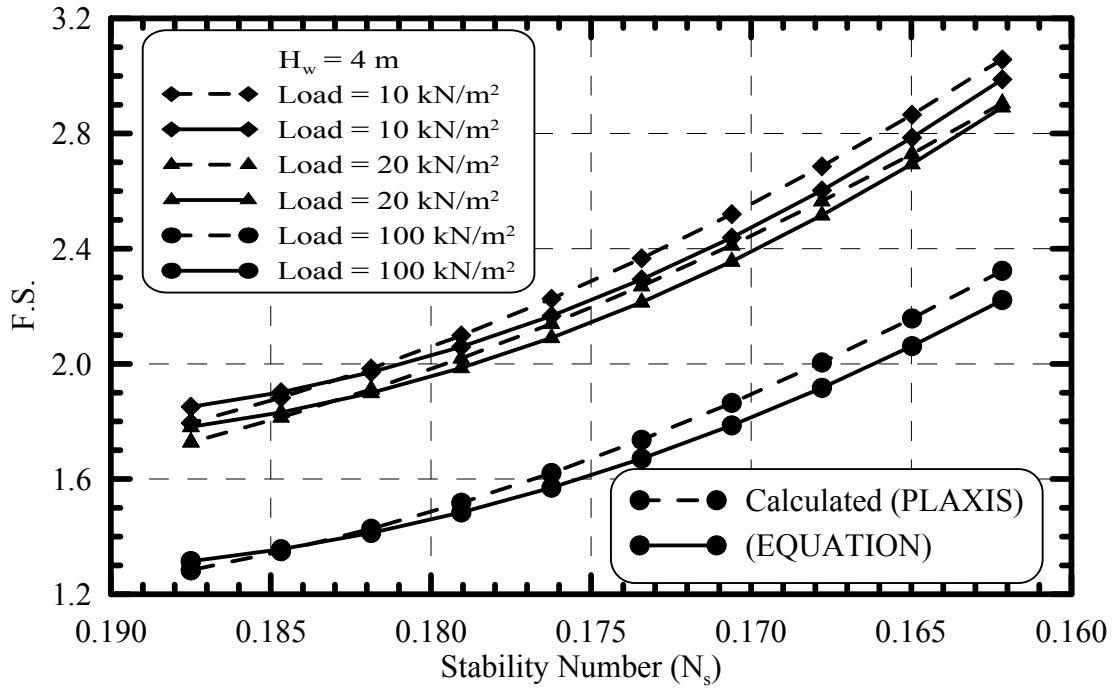


Fig. 12: Relation between the Stability Numbers (N_s) and (F.S.).

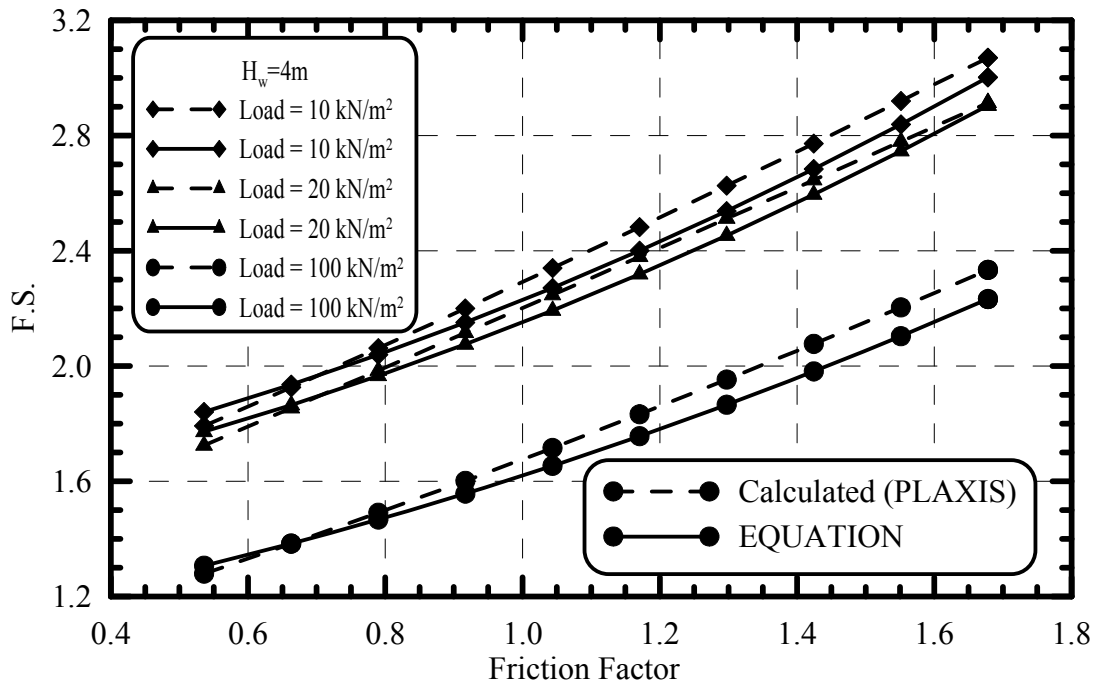


Fig. 13: Relation between the Friction Factors (F) and (F.S.).