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Effect Of Knee Braces On The Sway Buckling Loads Of Multi-Storey Frames

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Abstract:

In this paper the effect of providing knee braces on the stability of rectangular multi-storey frames with hinged and fixed bases is investigated. Two families frames provided with knee braces are studied. The effect of the various factors affecting the elastic buckling load is discussed.
The sway buckling loads are calculated using the direct method of analysis, which is based on the resolution of the general state of sway in the presence of axial compression. The results are represented diagrammatically as function of the buckling load with respect to the different effective parameters.

Keywords:

Buckling Loads, knee Braces, Bending Stiffness, Bracing Coefficient, Axial Deformation.

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1. Introduction:

In order to improve the strength of frameworks against lateral loads, the center lines of bracing members intersect the beams at a small distance from the column-to-beam connections. These braces may have different lengths and heights according to the required clearances governed by the machinery dimensions and the circulation inside the building. The effect of knee on the elastic sway critical loads of the rectangular multi-storey frames with hinged and fixed base is introduced in this paper. In fact, the presence of knee braces creates a resistance to lateral influence through a set of induced forces in the parts of columns and beams enclosed by bracing members. Two families of multi-storey frames provided with knee braces are studded. The first family 'family(A)' is fixed and hinged base frames having constant column and beam sections .The frame provided with constant cross sectional area of knee braces at both corners of each storey and is loaded only at the top, fig.(1-a). The second family 'family(B)' is fixed and hinged base frames having variable column sections and constant beam sections .The frame provided with constant cross sectional area of knee braces at both corners of each storey and is loaded at both corners of each storey and is loaded at intermediate floor levels, fig.(1-b). This paper is organized as follows: DIRECT METHOD OF ANALYSIS is presented in Section 2. Section 3 describes the MULTI-STOREY FRAMES PROVIDED WITH KNEE BRACESFAMILY (A). In Section 4 MULTI-STOREY FRAMES PROVIDED WITH KNEE BRACESFAMILY (B) is introduced. The final section concludes the paper.

2. DIRECT METHOD OF ANALYSIS:

To determine the elastic sway buckling loads, a direct method of analysis is used on the work. This method of analysis is based on a pre- study of the possible buckling modes; symmetric mode, anti- symmetric mode, and unsymmetrical mode, so as to find which of these yields the least buckling load. The frame is assumed acted upon by a small external influence that when removed; the frame will maintain its deflected configuration for the mode chosen. The operations of rotation and sway are built up for every member of the frame separately corresponding to the buckled configuration of the frame. Then the conditions of equilibrium and compatibility are satisfied. Since at the critical load there are no moments at the corners to keep the frame in its deflected position, the summation of the bending moments therein should be equal to zero. This provides the first set of equations which are as many equations as the number of joints. Another set of equations can be obtained from geometric characteristics of frame like relating the relative sways of the tow columns in every storey. Eliminating the indeterminate variables from these systems of equations, which may be done in a determinant form yields the buckling load equation? There are an infinite number of solutions which may verify this condition but the least one is aimed at. The solution of such determinant can only be solved by the method of trail-and-error. An electronic computer is an efficient mean for this technique.

3. MULTI-STOREY FRAMES PROVIDED WITH KNEE BRACES FAMILY (A):

3.1 EQUATIONS OF ELASTIC CRITICAL LOADS:

Consider single-bay multi-storey fixed (hinged) base frameworks provided with knee braces (axial deformations in bracing member are neglected and included):

3.1.1. Eccentricity ratio (b/B) within the interval from 0.0 to 0.5:

Consider the rectangular N –storey frame provided with knee braces at both corners of each storey. The frame has constant and column sections, and loaded at the top by two vertical panel loads P, figure (1-a). The height and width of the brace are equal for all stories. Since, the frame is symmetrical in loading and dimensions, the anti- symmetrical buckling mode is expected. The operations of rotation and translation are written for each member of intermediate floor panel referred to by NO. (J) Corresponding to the distorted configuration, fig. (2). There are no external moments at the joint to keep the frame in its deflected position; the sum of moments therein should be equal to zero. Thus: $\sum$ M (3J-2) =0;

$$-o1\frac{K1}{KC}\,Ø\,(3J-4) + \left(n1\frac{K1}{KC} + n2\frac{K2}{KC}\right) Ø\,(3J-2) - o2\frac{K2}{KC}\,Ø\,(3J-1) + \frac{m2}{2}\cdot\frac{F(J).a\sin\Phi}{KC} = 0; \qquad (1)$$

$\sum$ M (3J-1) =0;

$$-o2\frac{K2}{KC}\,Ø\,(3J-2) + \left(n1\frac{K1}{KC} + n2\frac{K2}{KC} + \frac{K3}{KC}\right) Ø\,(3J-1) - \frac{K3}{KC}\,Ø\,(3J) - o1\frac{K1}{KC}\,Ø\,(3J+1) + \frac{1}{2}\cdot\frac{V(J).b}{KC} + + \left(\frac{m2}{2} - \frac{1}{2}\right)\cdot\frac{F(J).a\sin\Phi}{KC} = 0; \qquad (2)$$

$\sum$ M (3J) =0;

$$-\frac{K3}{KC}\,Ø\,(3J-1) + \frac{K3}{KC}\,Ø\,(3J) + \left(\frac{1}{2} + \frac{B-2.b}{2.b}\right)\cdot\frac{V(J).b}{KC} - \left(\frac{1}{2}\right)\cdot\frac{F(J).a\sin\Phi}{KC} = 0; \qquad (3)$$

From anti-symmetry, Δ (3J-1) (3J) =Δ (3J) (3J)', then it may be written in another form;

$$\Delta(3J-1)\,(3J) = \frac{1}{2}\Delta(3J)\,(3J)'$$

$$\frac{\Delta(3J-1)\,(3J)}{b} = \frac{1}{2}Ø(3J-1) + \frac{1}{2}\,Ø\,(3J) - \frac{1}{12}\,\frac{kc.V(J).b}{k3\,.kc} + \frac{1}{12}\cdot\frac{kc.\;F(J).a\,.\cos\Phi}{k3\,.\;kc} \qquad (4\text{-}a)$$

$$\frac{\Delta(3J)\,(3J)'}{B-2b} = -\frac{1}{2}\,Ø\,(J) - \frac{1}{2}\,Ø(J) + \frac{1}{12}\cdot\frac{V(J).(B-2b)}{k4}$$

Multiplying both sides by (B-2b)/2b;

$$\frac{1}{2}.\frac{\Delta(3J)\,(3J)'}{b} = -\ \frac{1}{2}(\frac{B-2b}{b})\ Ø\ (J)+\frac{1}{24}\ \frac{Kc(B-2b)^{2}}{K4.(b^{2})}\frac{V(J).b}{Kc} \qquad (4\text{-}b)$$

Subtracting Eqn. (4-b) from Eqn. (4-a) and replacing the ratio $\frac{(B-2b)}{b}$ by the bending stiffness ratio ($\frac{K3}{K4}$);

$$\frac{1}{2}Ø(3J\text{-}1)\ +(\frac{1}{2}+\frac{1}{2}(\frac{B-2b}{b}))\ Ø(J)\text{-}\ \frac{1}{24}\frac{Kc}{K3}(2+(\frac{K3}{K4})^{3})\ \frac{V(J).b}{Kc}+\frac{1}{12}\frac{Kc}{K3}\ \frac{F\ (J).a\ .\cos\Phi}{Kc}\ =0 \qquad (4)$$

Since the axial deformations of the members are neglected, the angle of translation of member (3J-2) (3J-1), (Ω1), should be equal to the angle of translation of member $(3J-1)\ (3J)$,(Ω2). For small deformations, the angle of translation of any member may be expressed by the tangent of the angle, thus:

Ω1= Ω2

$$\frac{\Delta(3J-2)(3J-1)}{a}=\frac{\Delta(3J-1)\,(3J)}{b}$$

$$\frac{\Delta(3J-2)(3J-1)}{a}\ =\frac{m2}{2}Ø\ (3J\text{-}2)\ +\frac{m2}{2}Ø\ (3J\text{-}1)\ \text{-}\ \frac{m2}{2S2(1+C2)}\ \frac{F\ (J).a.\cos\Phi}{K2} \qquad (5\text{-}a)$$

Subtraction Eqn. (4-a) from Eqn. (5-a) then;

$$\frac{m2}{2}Ø(3J\text{-}2)+(\frac{m2}{2}-\frac{1}{2})Ø(3J\text{-}1)\text{-}\ \frac{1}{2}Ø(3J)+\ \frac{1}{12}\ \frac{Kc}{K3}\ \frac{V(J).b}{Kc}\text{-}$$

$$(\frac{m2}{2S2(1+C2)}\ \frac{Kc}{K2}+\frac{Kc}{K3})\ \frac{F(J).a.\cos\Phi}{Kc}\ =0; \qquad (5)$$

Where; $Kc=\frac{EI}{L}$;

Kb=EIb/B;

K1=EIC/ (L-a); (Constant for all stories)

K2=EIC/a; (Constant for all stories)

K3=EIb/b;

K4=EIb/ (B-2b);

<u>3.1.2. Eccentricity ratio (b/B) within the interval from 0.5 to 1.0:</u>

Following the same analytical procedure used before translation for each part Applying the five previously used conditions except that angle of translation of member $(3J-1)\ (3J)$,(Ω2) , is calculated from a different manner;

$$Ω2=\frac{\Delta(3J-1)\,(3J)'}{b}\ =\ \frac{\Delta(3J)\,(3J)'}{b}\ \text{-}\ \frac{\Delta(3J-1)\,(3J)}{b}$$

K3=EIb/ (B-b);

K4=EIb/ (2b-B);

$$\rho c = \frac{P.L}{\pi^2 \,\mathrm{Kc}} \;;$$

$$\rho 1 = \frac{P.(L-a)}{\pi^2 \,\mathrm{K1}} = \rho c.\left(\frac{L-a}{L}\right)^2 ,$$

$$\rho 2 = \frac{P.a}{\pi^2 \,\mathrm{K2}} = \rho c.\left(\frac{a}{L}\right)^2 ,$$

3.2. Effect of axial Deformations in Braces on Elastic Critical Loads:

Reconsider the rectangular frame shown in Figure (1). When the axial deformations of the braces are considered, all the equations in Sec. (3.1) will be valid except Eqn. (5). Focuses on the swayed triangle in Figure (3) which includes the left column, the L.H.S. part of beam, and the left bracing member binding them. From Figure (3), the following relation can be deduced:

$$\Delta(3J-2)(3J-1).\cos\Phi \;=\Delta\,(3J-1)(3J)\sin\Phi \;+ \mathrm{e}$$

Where (e) the axial deformation developed in the bracing member.

Divided the two sides by (a. $\cos\Phi$):

$$\frac{\Delta(3J-2)(3J-1)}{a} - \frac{\Delta(3J-1)\,(3J)}{a}\tan\Phi = \frac{e}{\mathrm{L}.\cos\Phi} \qquad (6)$$

Since tan Φ= a/b, then substituting in Eqn. (6):

$$\frac{\Delta(3J-2)(3J-1)}{a} - \frac{\Delta(3J-1)\,(3J)}{b} = \frac{e}{a.\cos\Phi} \qquad (7)$$

The left side of this equation is the same as in Eqn. (5). Carrying out some operations on the term $\frac{e}{\mathrm{a}\;\cos\Phi}$ to be:

$$\mathrm{E} = \frac{stress}{strain} = \frac{F(J)}{\mathrm{Abrac}} \div \frac{e}{a/\sin\Phi} \qquad (8.a)$$

$$\frac{e}{\mathrm{a}.\cos\Phi} = \frac{F(J)}{\mathrm{Abrac.E.sin}\,\Phi.\,\cos\Phi}$$

$$= \frac{IC}{\mathrm{Abrac.a^2.sin}\,\Phi.\cos\Phi^2}.\frac{F(J).\mathrm{a}.\cos\Phi}{K2}$$

$$= \frac{1}{\beta}.\frac{F(J).\mathrm{a}.\cos\Phi}{K2} \qquad (8.b)$$

Then Eqn. (5) gets the form:

$$\frac{m2}{2}\;\text{Ø}\;(3J\text{-}2)\;+\left(\frac{m2}{2}-\frac{1}{2}\right)\;\text{Ø}\;(3J\text{-}1)\;-\;\frac{1}{2}\;\text{Ø}\;(3J)\;+\frac{1}{12}\frac{\mathrm{Kc}}{.\mathrm{K3}.}\frac{\mathrm{V(J).b}}{\mathrm{Kc}}\;-\;\left[\frac{m2}{2S2(1+C2)}\frac{\mathrm{Kc}}{\mathrm{K2}}+\frac{1}{12}\frac{\mathrm{Kc}}{\mathrm{K3}}+\frac{1}{\beta.k2c}\right]\frac{\mathrm{F(J).a.cos}\,\Phi}{\mathrm{Kc}}=0 \qquad (8)$$

Where $\beta = \frac{Abrac\,.L^2.\sin \Phi.\ \cos^2 \Phi}{IC}$

The problem contained six variables:

1-The bending stiffness ratio Kb/Kc;

2-The knee width to the beam length ratio b/B;

3- The knee height to the column height ratio a/L;

4- The knee bracing stiffness coefficient β (bitta);

5- The vertical load to the column's Euler load ratio,

6- The number of stories,

The results of the critical buckling load for the frame (family-A) are illustrated in Figure (4) and (5) for different and practical values of other variables.

4. MULTI-STOREY FRAMES PROVIDED WITH KNEE BRACES FAMILY (B):

The frame has constant beam sections and variable column sections, and loaded at intermediate floor levels by vertical panel loads P/N,fig.(1-b). The height and width of the brace are equal for all stories. Since, the frame is symmetrical in loading and dimensions, the anti- symmetrical buckling mode is expected. The operations of rotation and translation are similar as the previous frame, expect that:

K1 (J) =K1*$(\frac{N+1-J}{N})$;

K2 (J) =K2*$(\frac{N+1-J}{N})$;

Following the same procedure applied in section (III). The problem contained the same previous variables which are presented in section(III).The results of the critical buckling load for the frame are illustrated in figs.(6)and(7) for different and practical values of other variables.

5. CONCLUSION:

The following conclusions can be deduced from this study:

1- The buckling loads increase rapidly with the increase of ratios (a/L) and (b/B) up to b/B=0.5. Beyond this specific ratio, each curve tends to have nearly a constant value which is the sway buckling load of the knee braced frame with the same bending stiffness ratio Kb/Kc. This means that, there is no tangible influence of ratio on the sway buckling loads when the ratio b/B exceeds 0.5.

2-Frames with smaller bending stiffness ratios have bigger improvements in the sway buckling loads with the increase of ratios b/B and a/L in the range less than 0.5, for b/B,

3-Also from these curves ,it may be noticed be that, the sway buckling loads at b/B equal 1.0 are the same as for b/B equal 0.5 for all values of Kb/Kc and a/L and represent anti-symmetric no-sway buckling loads.

4-The buckling loads (when axial deformations are neglected) for frames with bending stiffness Kb/Kc less or equal to 4.0 decrease mildly down to reach the least values of the buckling loads which generally lie between b/B=0.6 and b/B=0.8.Beyond these ratios, the buckling loads have a slight increase the case of cross- braced frames when b/B=1.0.

5-The maximum sway critical loads are almost found within a ratio(b/B) from the corners which ranges between0.3 to 0.4 for fixed base frames while for hinged base frames, this range is from 0.3 to 0.5,(when axial deformations are neglected).

6- The maximum sway critical loads are almost found within a ratio(b/B) from the corners which ranges between0.7 to 0.9 for fixed and hinged base frames,(when axial deformations are included).

7-With the increase of the number of stories, the buckling loads of multi-storey knee braced frames decreases.

8- For any number of stories in hinged base knee braced multi-storey frames of (A),the buckling load of first storey always governs the buckling of the whole frame when keeping the same bracing area and the same ratio(b/B) and(a/L) all through the frame height.

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7. LIST OF NOTATIONS:

A_{brac}- Cross- sectional area of knee brace member;

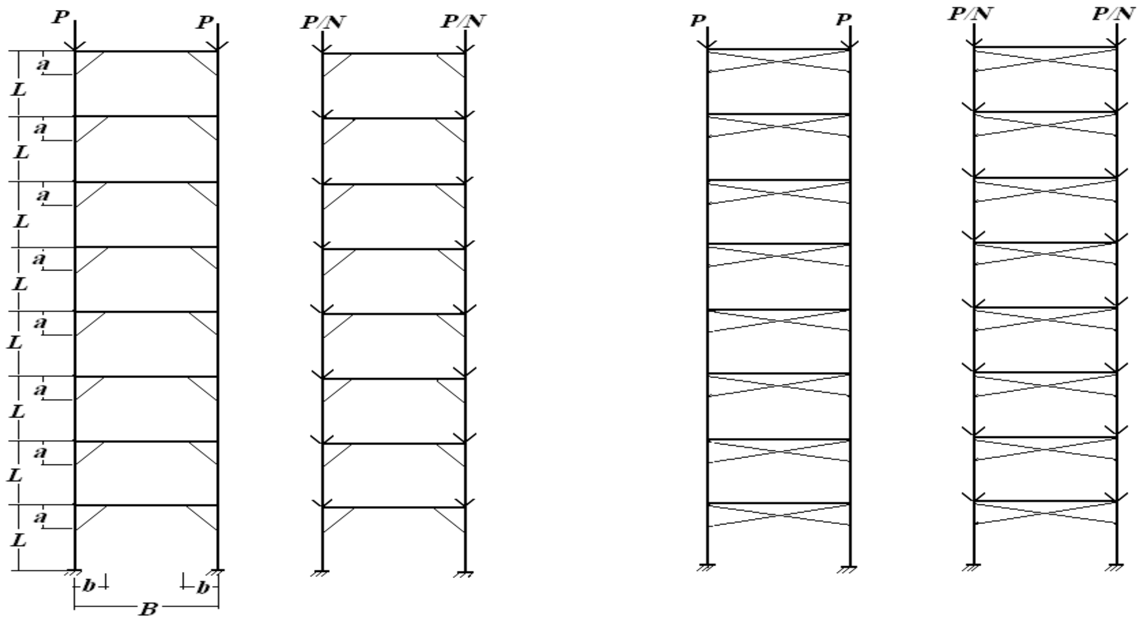
a-Knee brace height;

b- Knee brace width;

L- Height of column;

B- Span of bay;

I_C- Moment of inertia of frame column;
I_b- Moment of inertia of frame beam;
K_c- Bending stiffness of frame column;
K_b- Bending stiffness of frame beam;
P_{cr}- Elastic critical load;
Pe- Euler's load of column;
ρ_c- Critical load parameter;
β- Stiffness of knee brace;
Φ- Angle of inclination of a knee brace member with the beam;
F- Secondary axial force in a knee brace member;
V- Secondary vertical force produced in column due to side sway;
Ø- Angle of rotation;
Δ- Relative translation of member;
n- Carry-over factor for fixed end member which is in state of no-shear sway;
m- Magnification factor for moments produced at ends of fixed member which is a state of pure-shear sway due to axial force effect;
c- Carry- over factor for fixed member when side sway is prevented;
s- Stiffness factor for fixed end member when side sway is prevented.



(a) **(b)** **(a)** **(b)**

Fig. (1) The studied families of single bay-multi storey frames

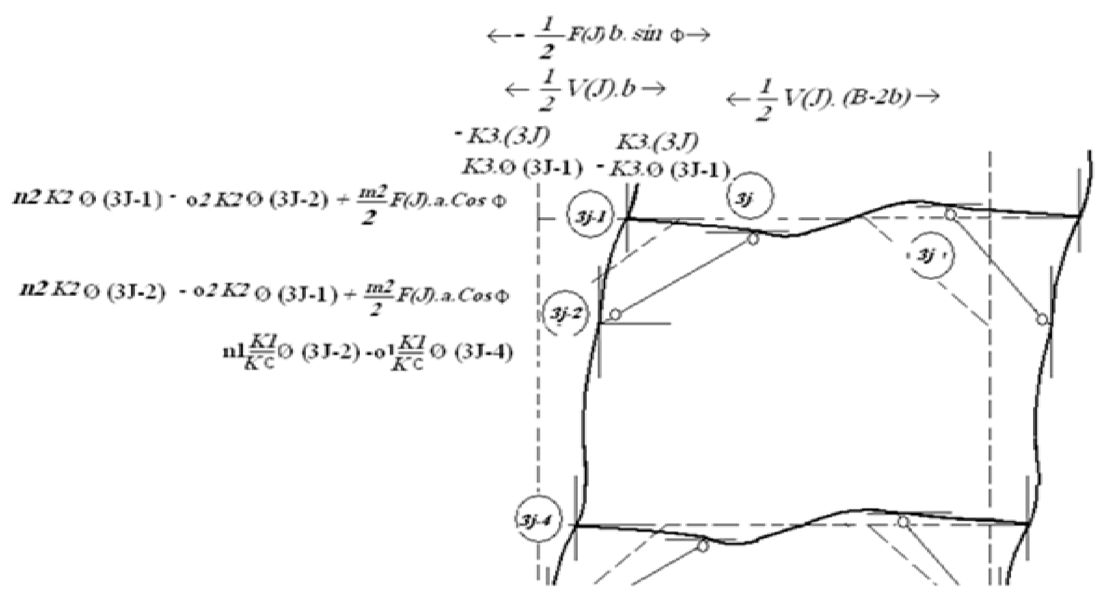


Fig. (2) Anti-symmetrical buckling mode and operations of rotation and sway of intermediate storey no."J".

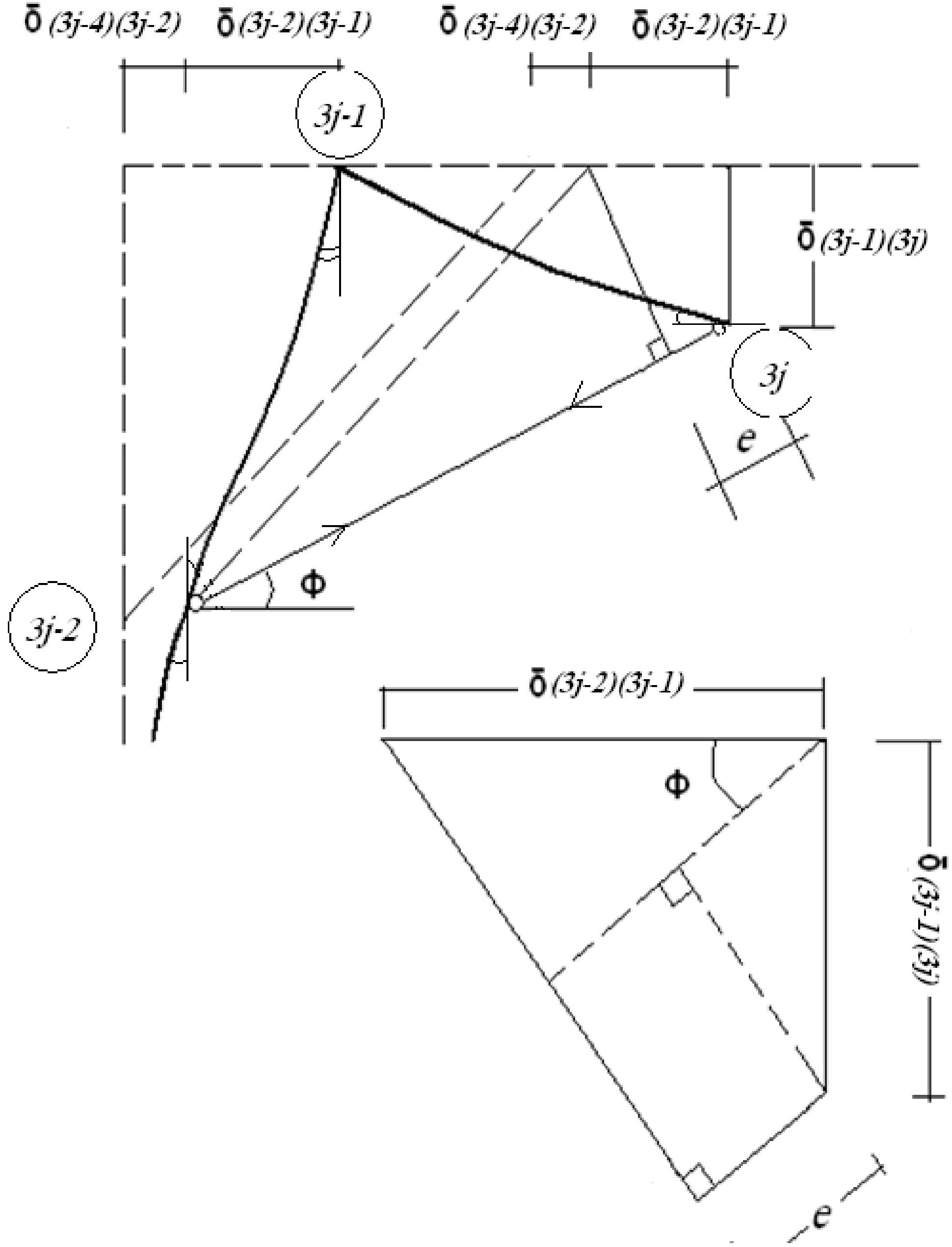


Fig. (3) Correlation between the joint displacements and axial deformation in bracing member

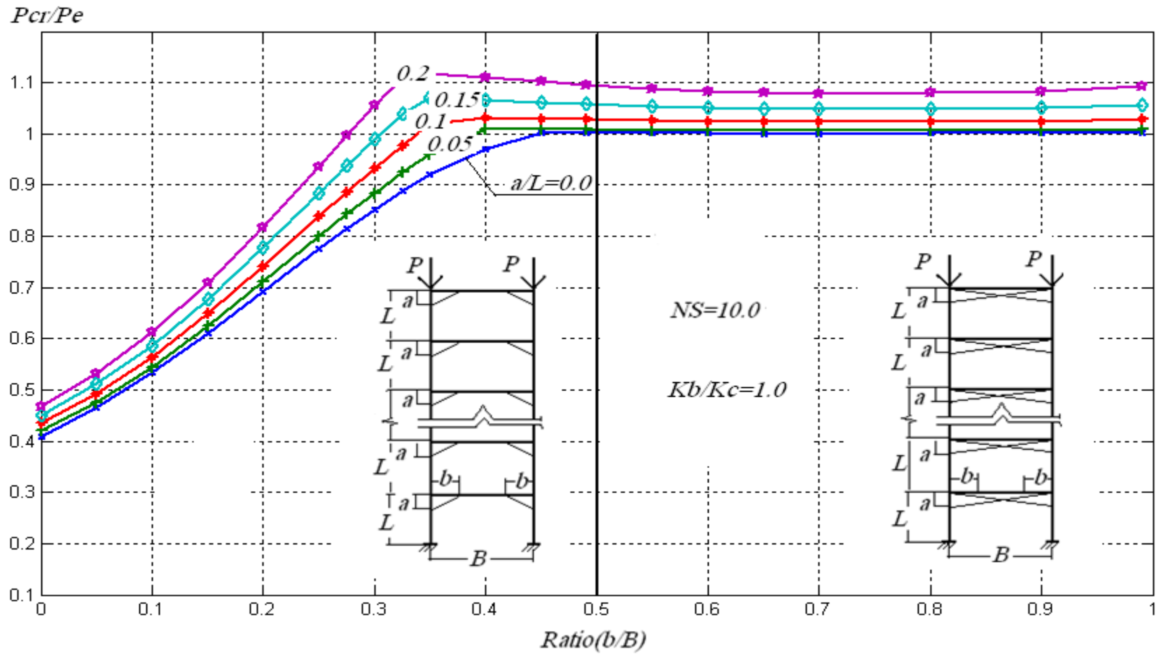


Fig. (4-a) Effect of the ratios (b/B) and ratio (a/L) on sway critical loads for fixed base frames (family -A)

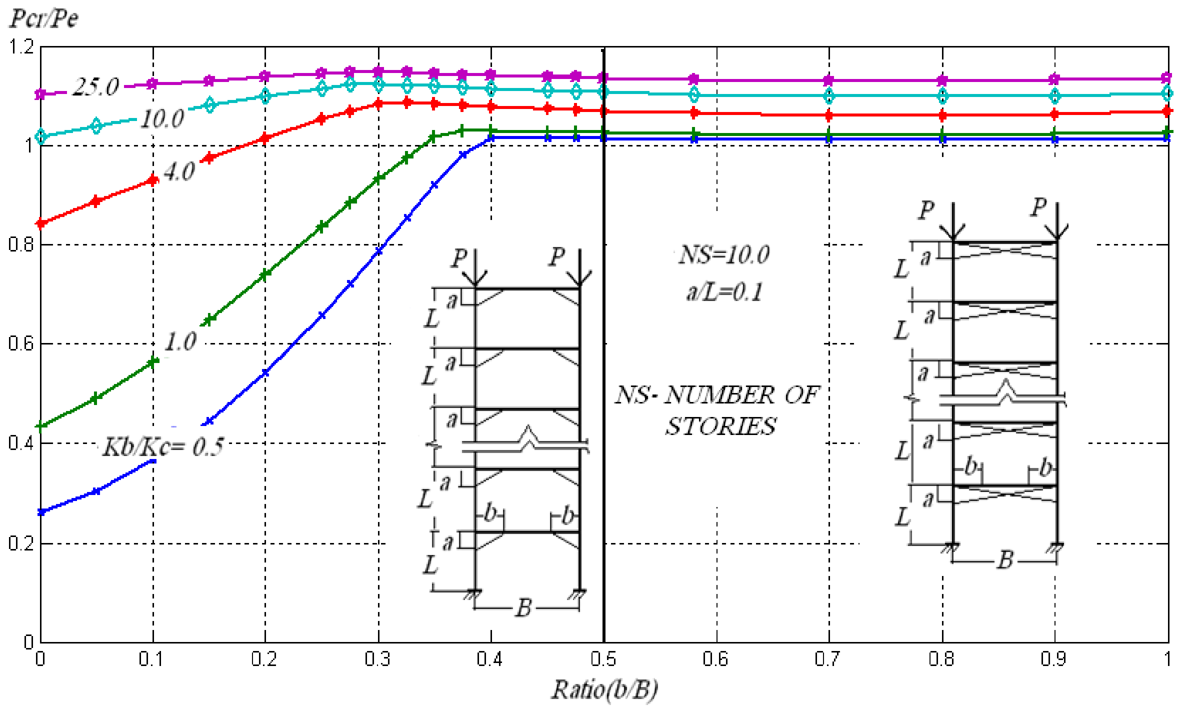


Fig. (4-b) Effect of the ratios (b/B) and ratio (a/L) on sway critical loads for fixed base frames (family- A)

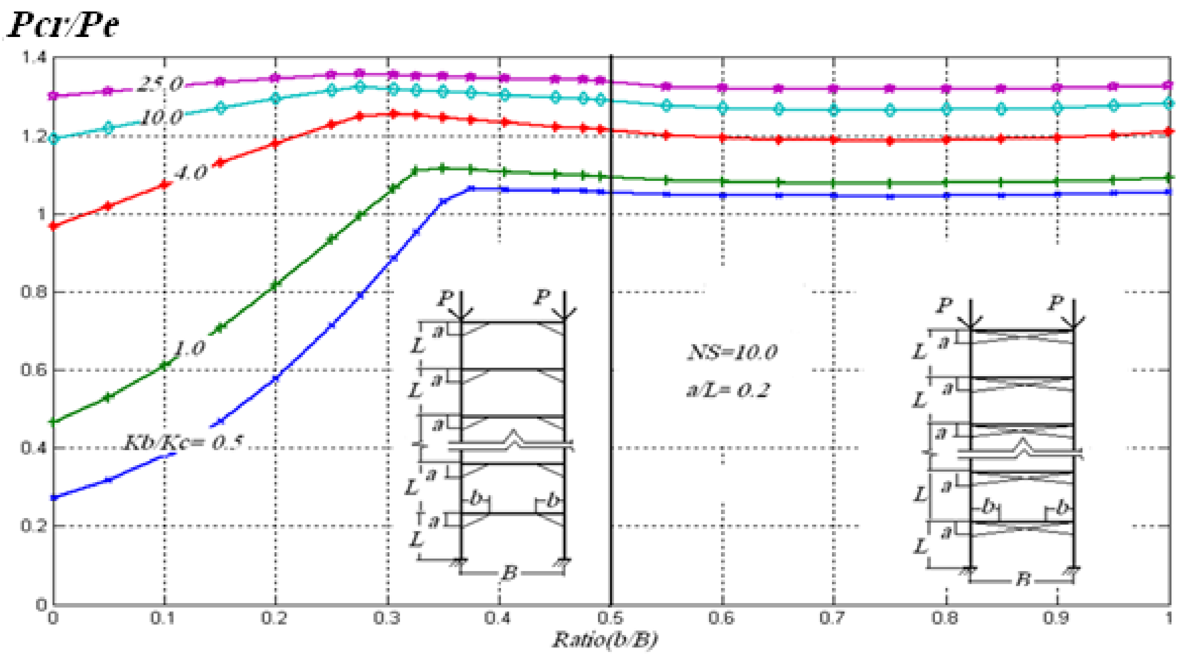


Fig. (4-d) Effect of the ratios (b/B) and ratio (a/L) on sway critical loads for fixed base frames (family- A)

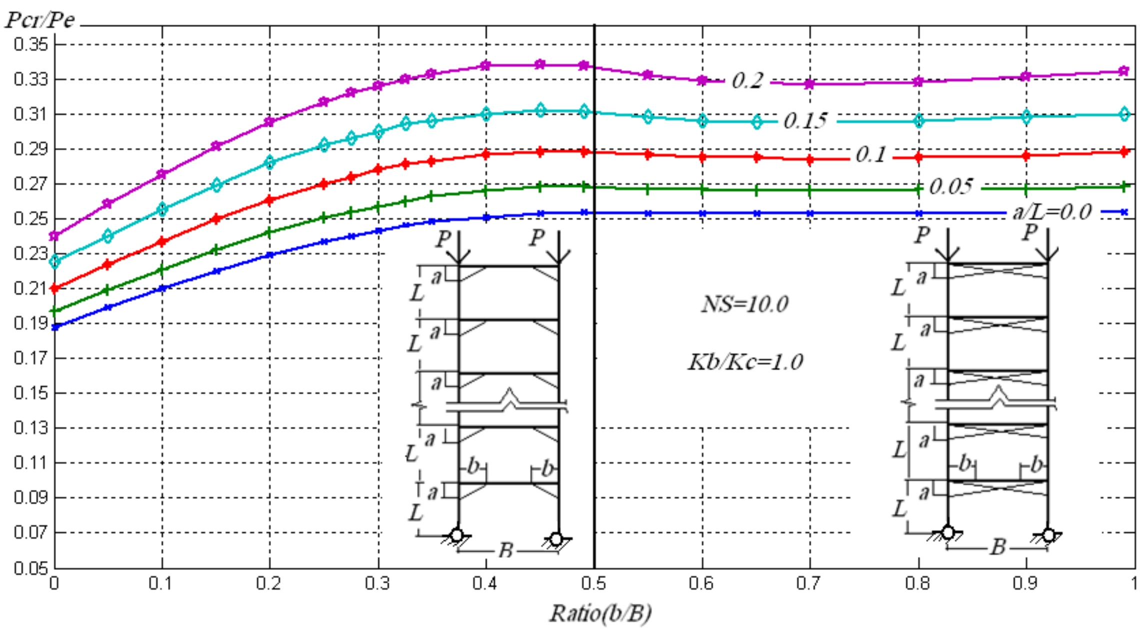


Fig. (4-e) Effect of the ratios (b/B) and (a/L) on sway critical loads for hinged base frames (family- A)

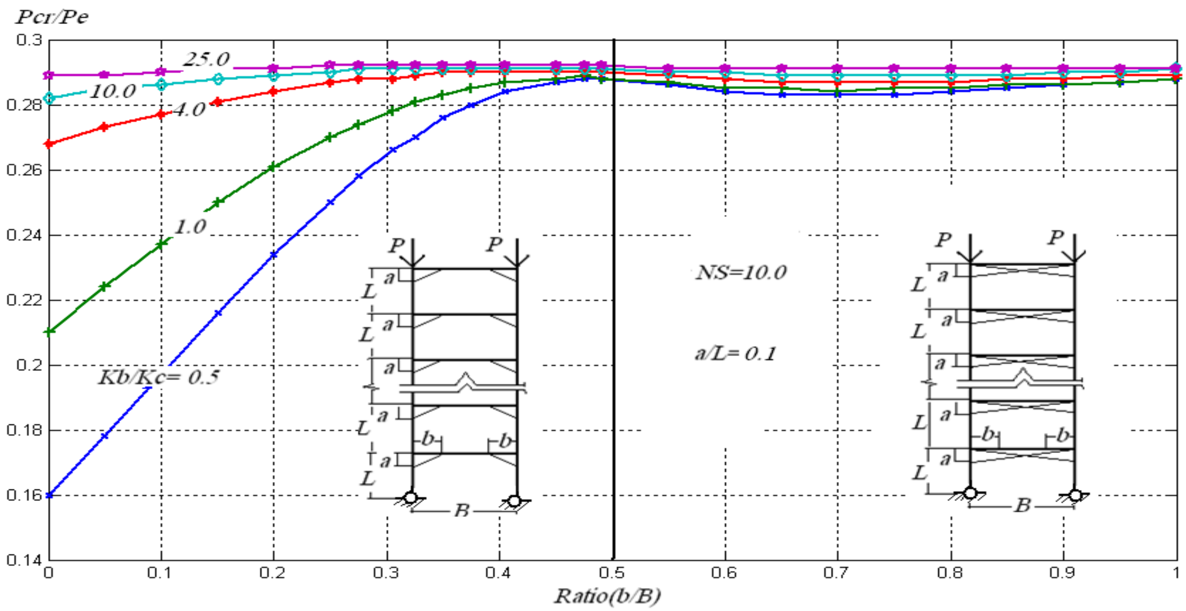


Fig. (4-f) Effect of the ratios (b/B) and (a/L) on sway critical loads for hinged base frames (family- A)

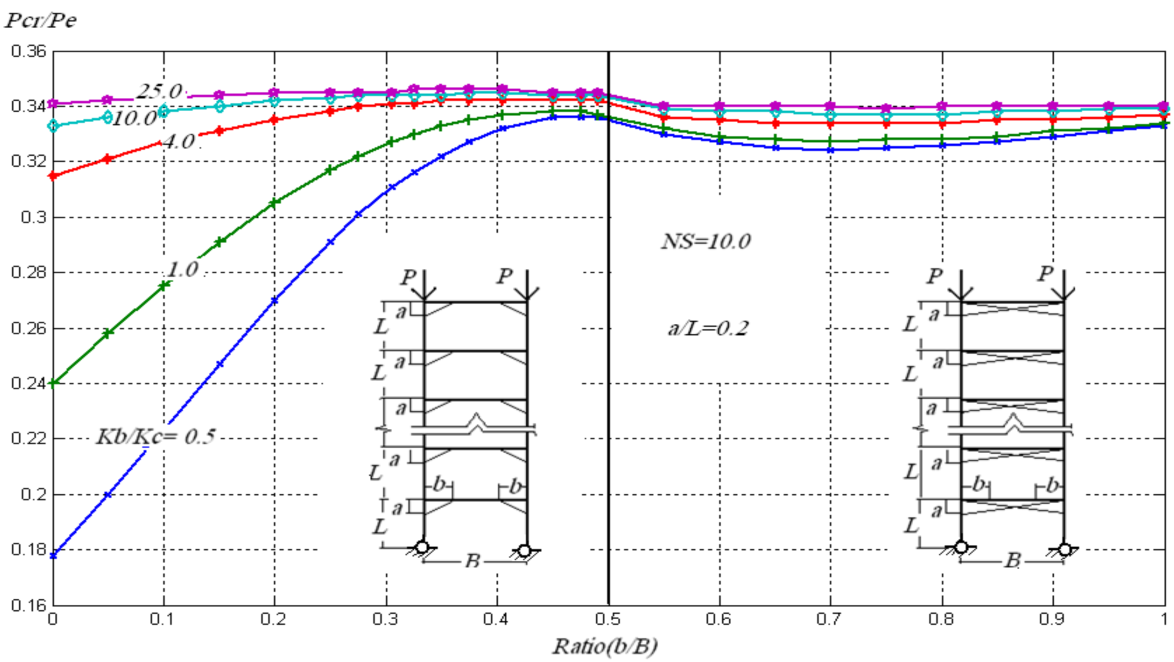


Fig. (4-g) Effect of the ratios (b/B) and (a/L) on sway critical loads for hinged base frames (family- A)

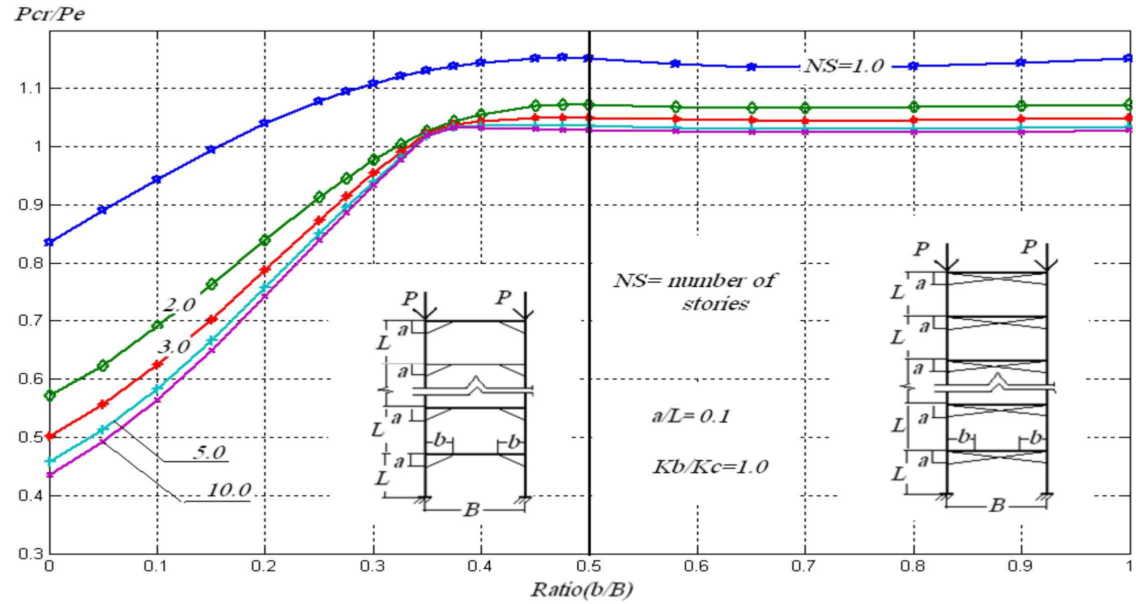


Fig. (4-k) Effect of the ratios (b/B), (a/L) and number of stories on sway critical loads for hinged base frames (family- A)

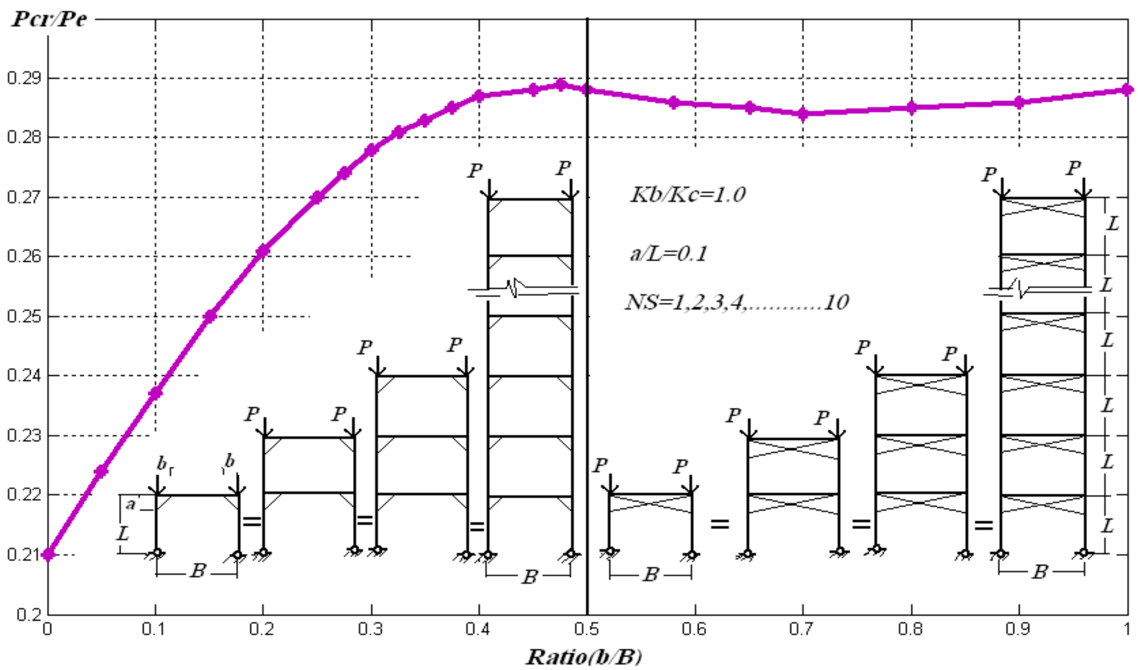


Fig. (4-m) Effect of the ratios (b/B), (a/L) and number of stories on sway critical loads for hinged base frames (family- A)

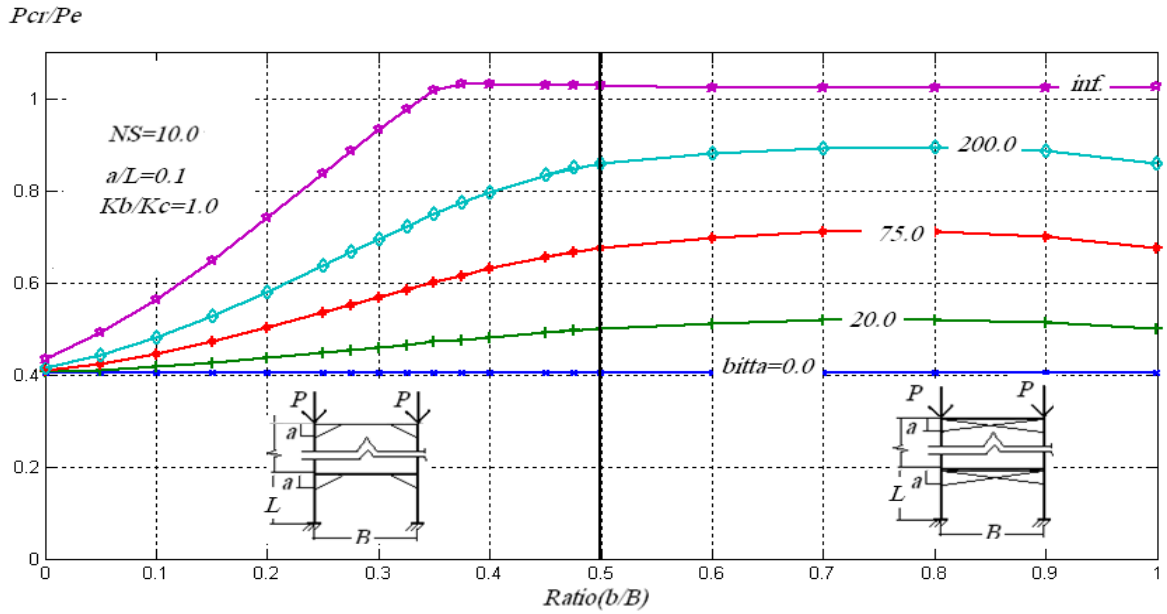


Fig. (5-a) Effect of the ratios (b/B) and (a/L) on sway critical loads for fixed base Frames (family- A) (axial deformations are included)

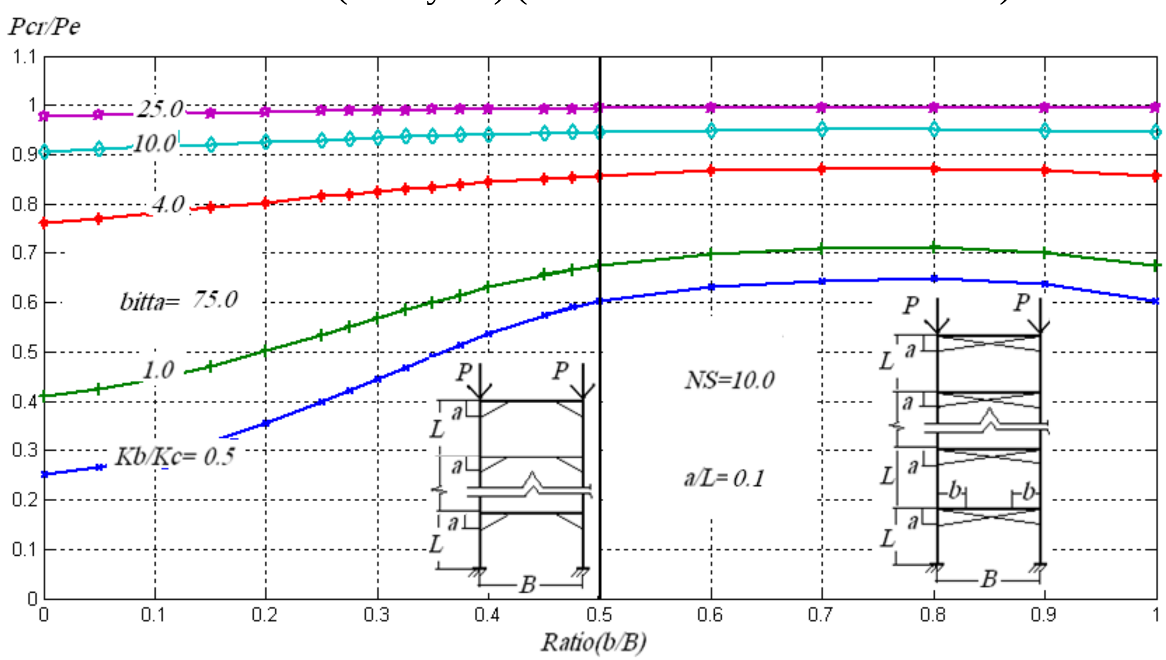


Fig. (5-b) Effect of the ratios (b/B) and (a/L) on sway critical loads for fixed base frames (family- A) (axial deformations are included)

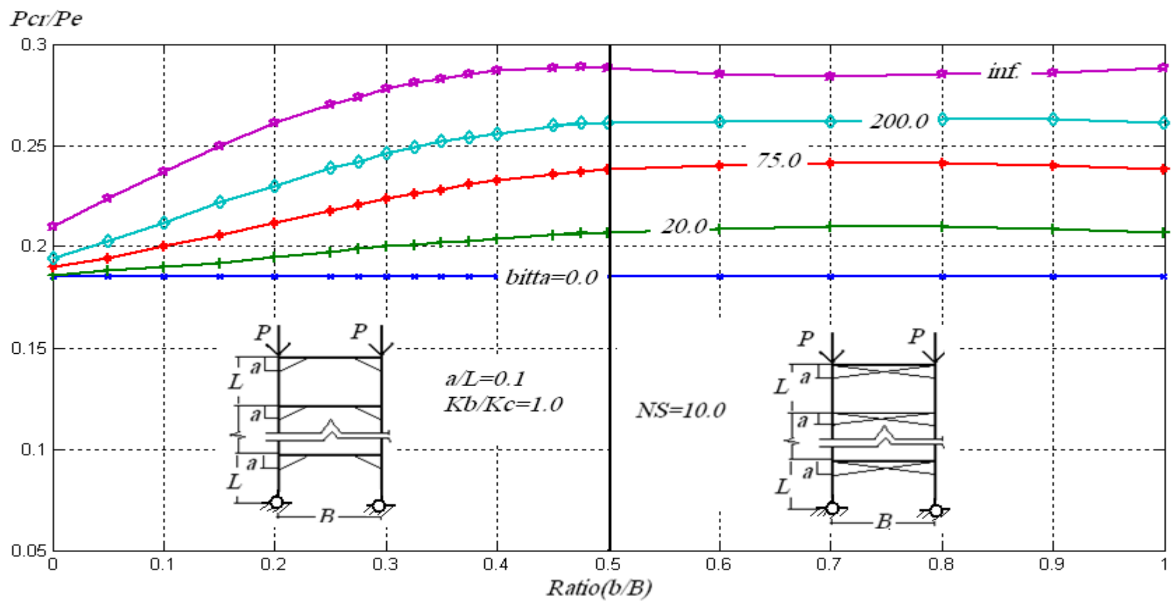


Fig. (5-c) Effect of the ratios (b/B) and (a/L) on sway critical loads for hinged base frames (family- A) (axial deformations are included)

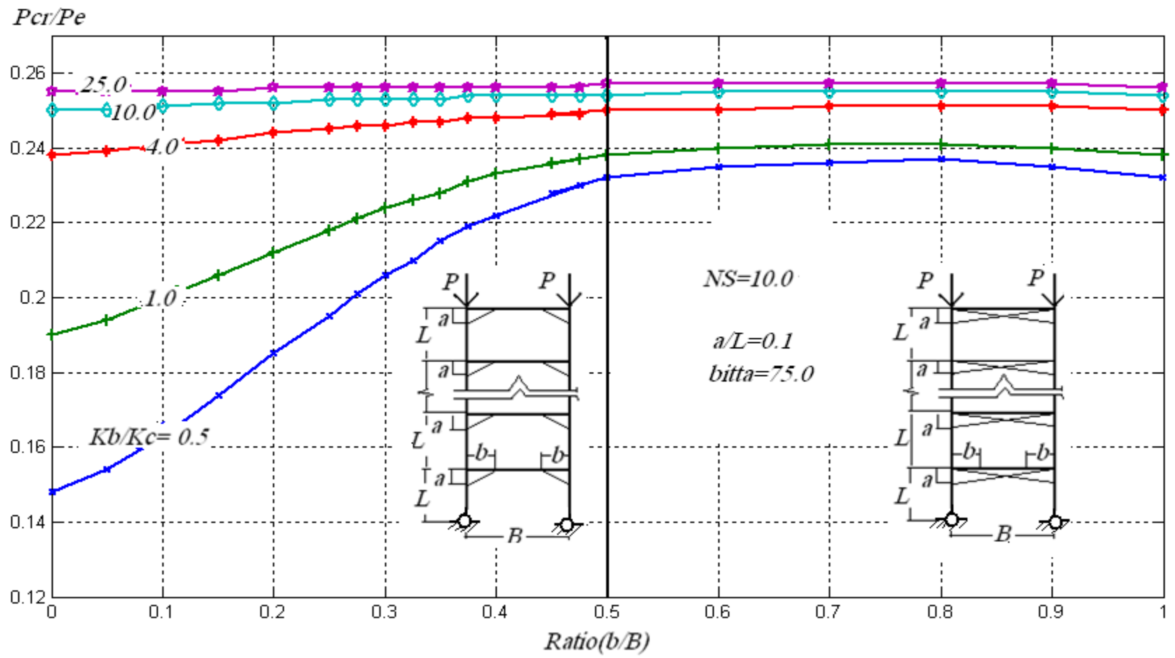


Fig. (5-e) Effect of the ratios (b/B) and (a/L) on sway critical loads for hinged base frames (family- A) (axial deformations are included)

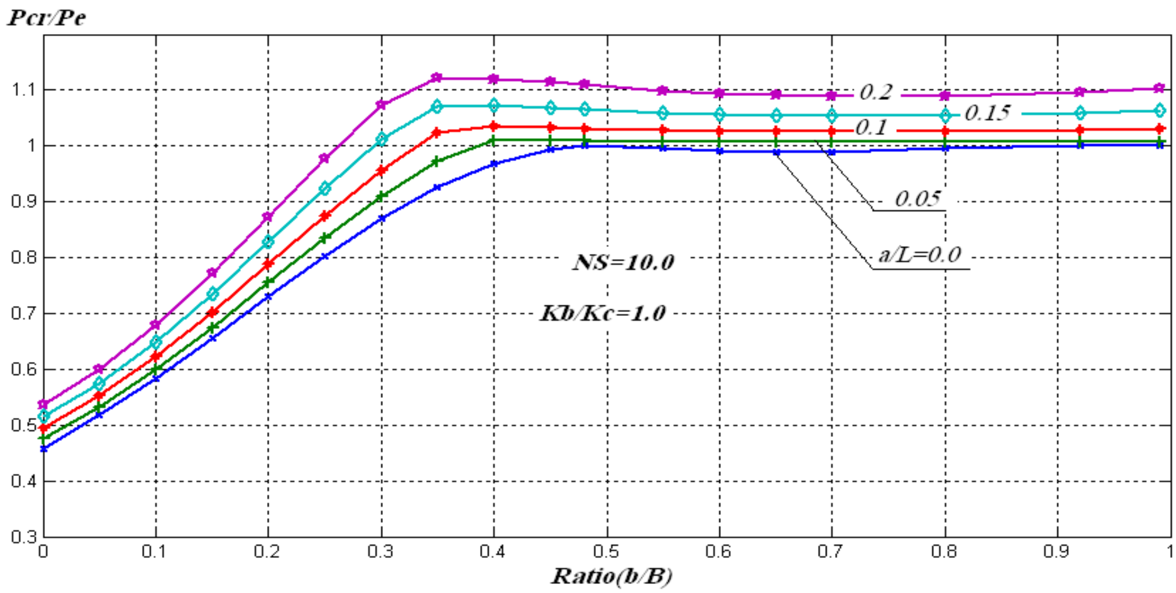


Fig. (6-a) Effect of the ratios (b/B) and ratio (a/L) on sway critical loads for fixed base frames (family- B)

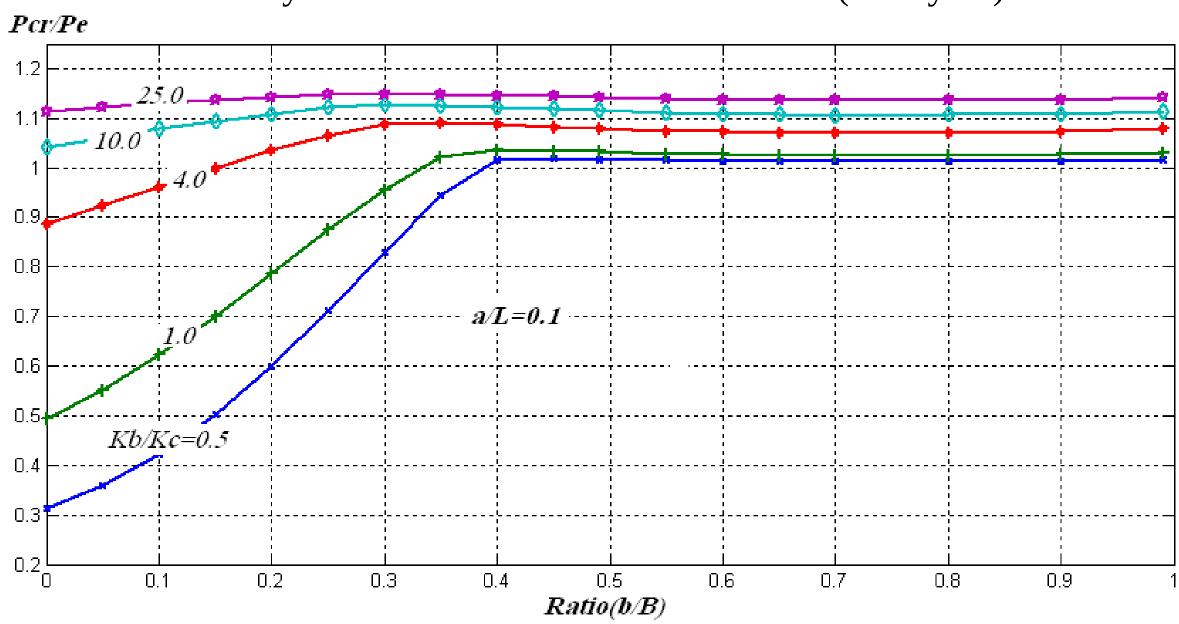


Fig. (6-b) Effect of the ratios (b/B) and ratio (a/L) on sway critical loads for fixed base frames (family- B)

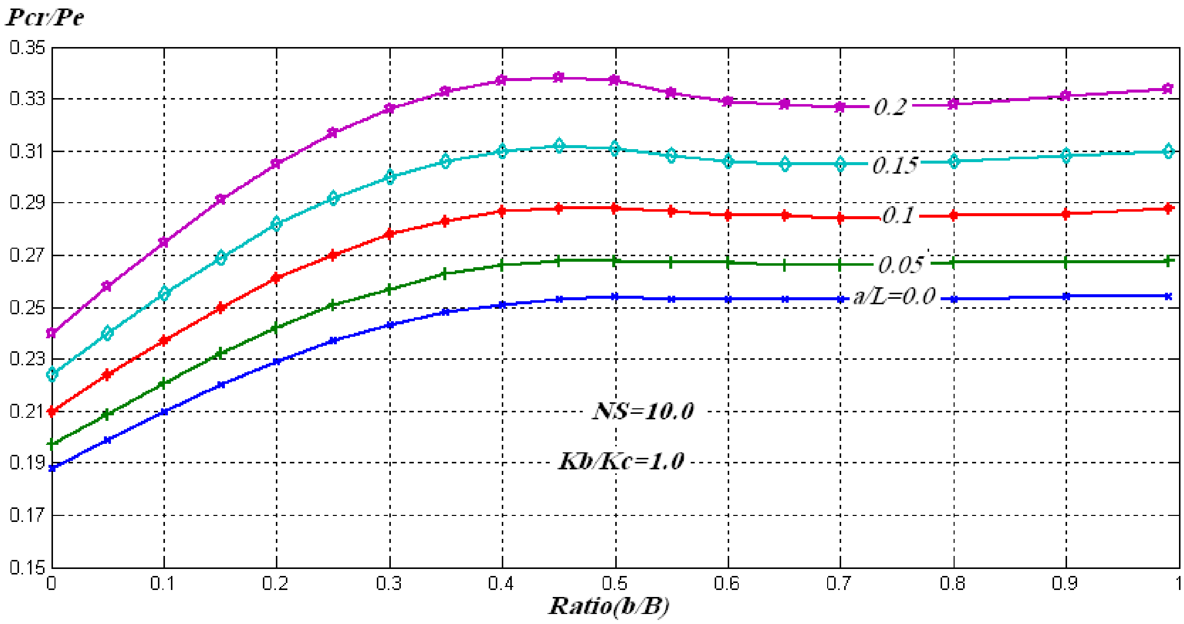


Fig. (6-c) Effect of the ratios (b/B) and ratio (a/L) on sway critical loads for hinged base frames (family- B)

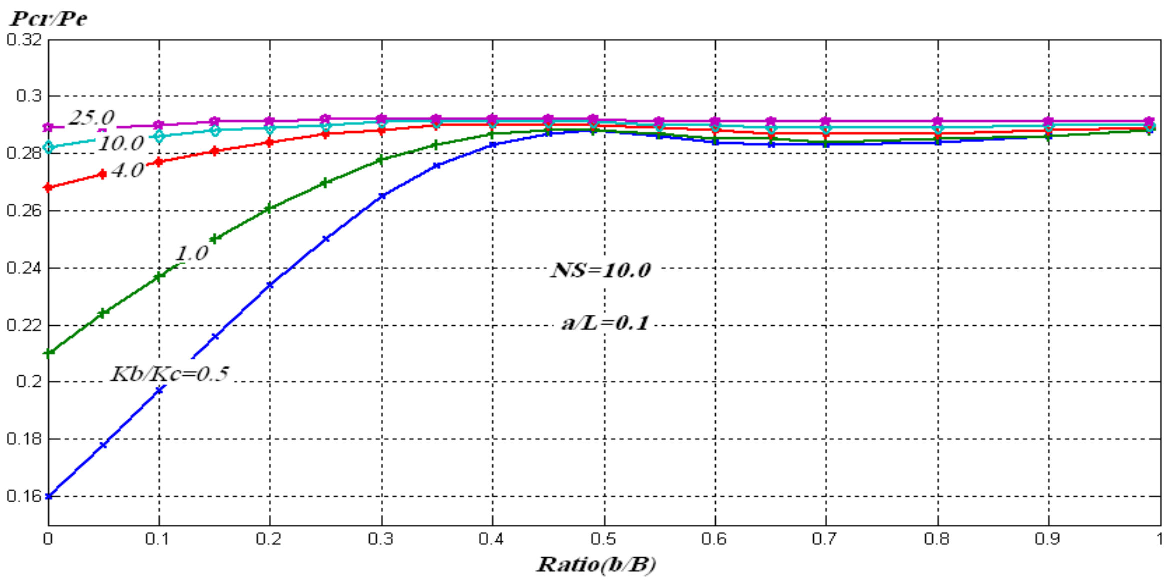


Fig. (6-d) Effect of the ratios (b/B) and ratio (a/L) on sway critical loads for hinged base frames (family- B)

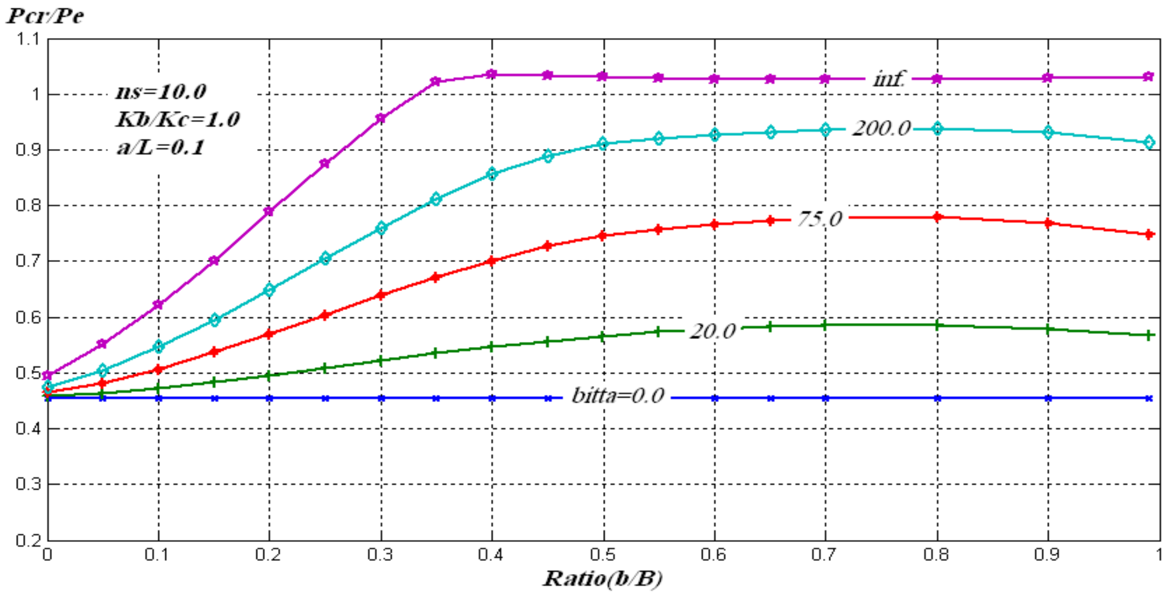


Fig. (7-a) Effect of the ratios (b/B) and ratio (a/L) on sway critical loads for fixed base frames (family- B) (axial deformations are included)

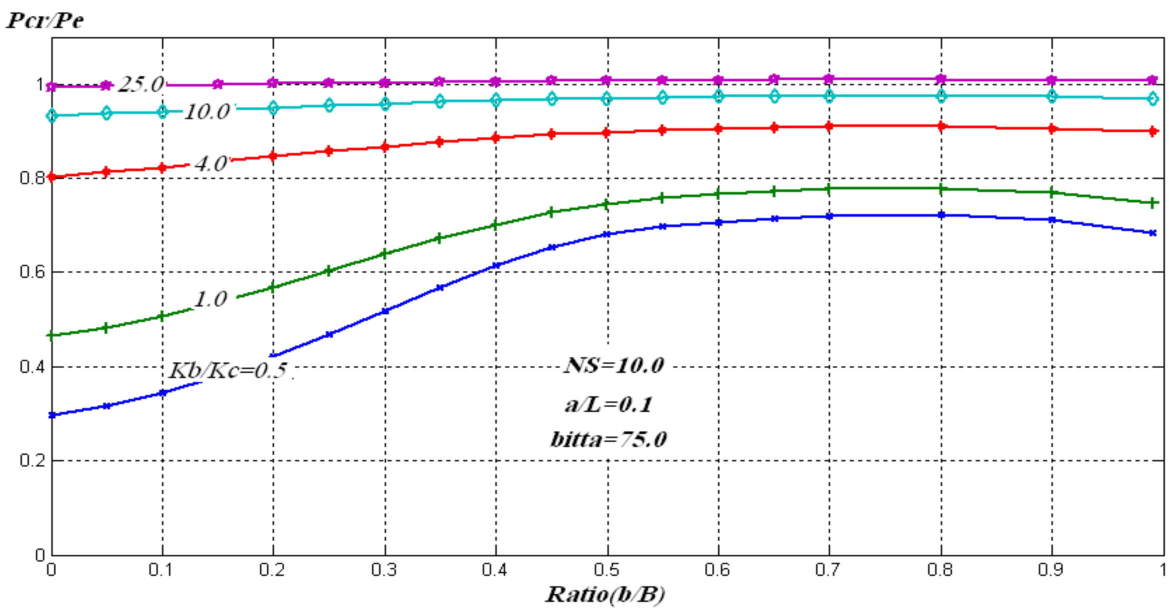


Fig. (7-b) Effect of the ratios (b/B) and ratio (a/L) on sway critical loads for fixed base frames (family- B) (axial deformations are included)

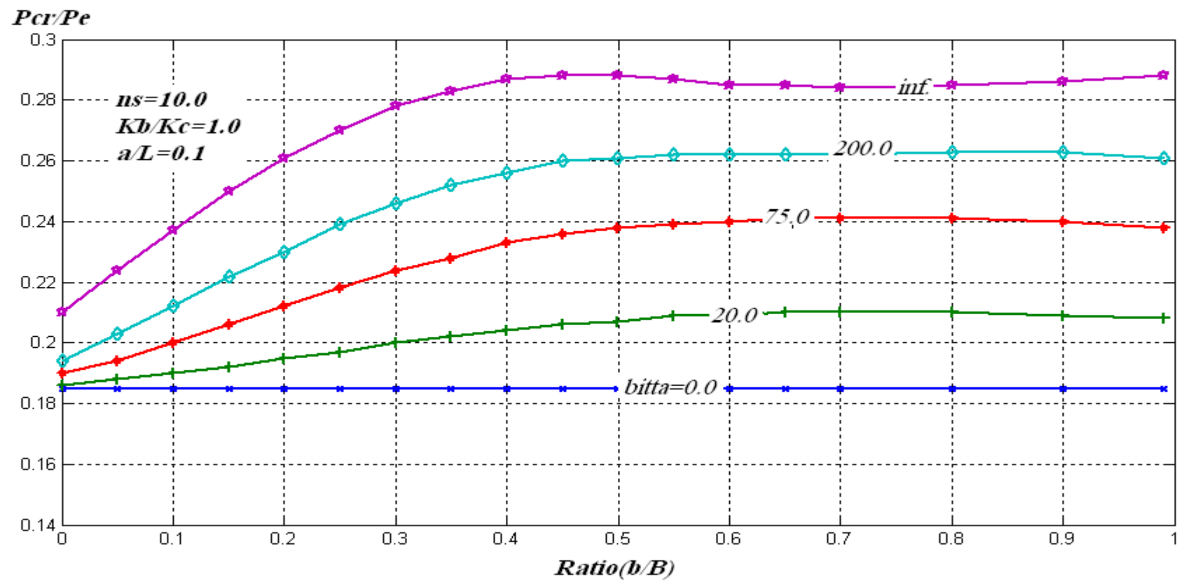


Fig. (7-c) Effect of the ratios (b/B) and ratio (a/L) on sway critical loads for hinged base frames (family- B) (axial deformations are included)

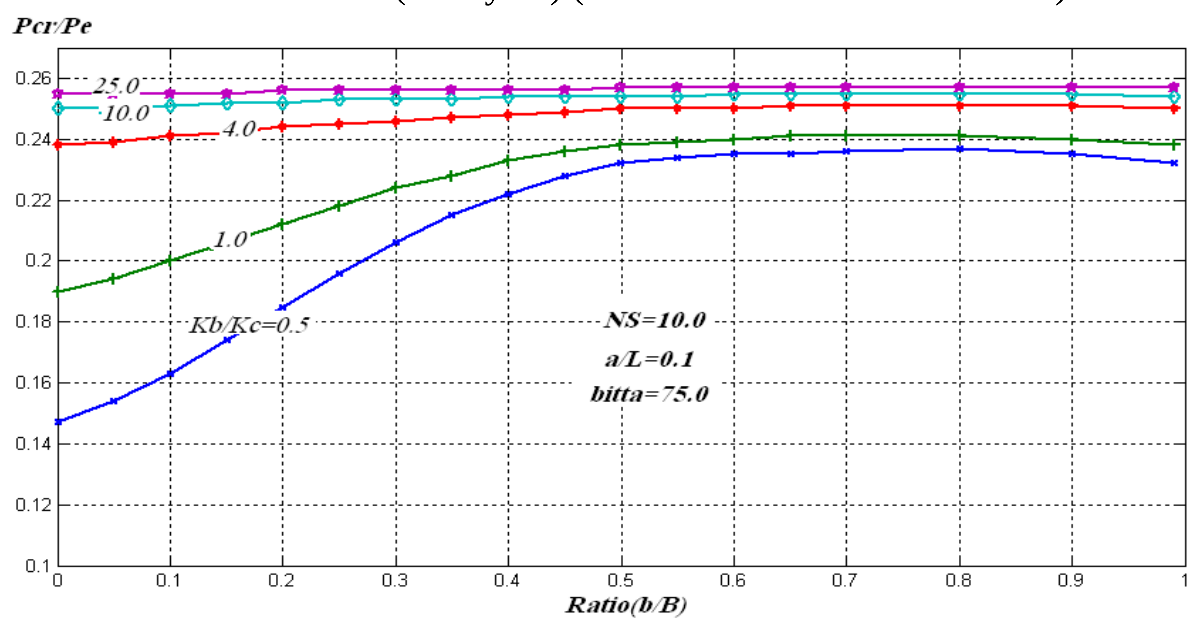


Fig. (7-d) Effect of the ratios (b/B) and ratio (a/L) on sway critical loads for hinged base frames(family- B) (axial deformations are included)