Proceedings of the 8th ICCAE-8 Conference, 25-27 May, 2010

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Military Technical College
Kobry El-Kobbah,
Cairo, Egypt

8th International Conference
on Civil and Architecture
Engineering
ICCAE-8-2010

A PROPOSED PROCEDURE FOR ANALYSING REINFORCED EMBANKMENTS

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Abstract

In this paper proposed equations are derived to calculate the factor of safety in addition to the force required for reinforcement to achieve the desired factor of safety in reinforced embankments. The equations are derived based on limit equilibrium approach. Assuming the failure surface to be an arc of a circle, solutions have been developed to take the effect of applied loads and the effect of applying the reinforcement in layers into account. A typical example has been analyzed to illustrate the use of the solutions. A comparison of the results is made with the results of Bishop's simplified method of slices through the software PROKON, and good convergence was obtained.

Key Words: Reinforced, Embankment, Stability, Slope.

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طريقة مقترحة لتحليل السداد الترابية المسلحة

الخلاصة:

في هذا البحث أشتقت معادلات مقترحة لحساب معامل الأمان بالإضافة إلى القوة المطلوبة في التسليح لتحقيق معامل الأمان المطلوب في السداد الترابية المسلحة. و قد أشتقت المعادلات استنادا إلى مبدأ التوازن الحدي. و قد طورت الحلول على افتراض أن سطح الفشل عبارة عن قوس دائرة و لتأخذ بنظر الاعتبار تأثير الأحمال المسلطة و أخذ توزيع التسليح على طبقات بالاعتبار أيضا. و قد تم تحليل مثال نموذجي لتوضيح استعمال المعادلات.

و أجريت مقارنة للنتائج مع نتائج طريقة الشرائح المبسطة لبيشوب من خلال برنامج PROKON و قد وجد تقارب جيد بين الطريقتين.

Introduction and scope:

The rotational stability often governs the design of reinforced embankments on soft soils. The maximum required reinforcement force P_{max}, to achieve the target factor of safety F, is usually calculated by limit equilibrium method and total stress analysis.

Low et al. (1990) presented the solutions in the form of charts and equations. The overall minimum factor of safety can be obtained by considering different limiting tangents, (Kaniraj and Abdullah, 1992 a).

Rotational stability:

Embankment and reinforcement details:

Fig .1 shows the details of a reinforced embankment of height H on a soft soil deposit. The height of tension crack in the embankment is H_c. The value of H_c may vary from 0 (no tension crack) to H (full height tension crack). The tension crack is assumed dry. The embankment has a stabilizing berm, the dimensions of which are expressed in terms of the height of embankment. The height and width of berm are k_1H and k_2H, respectively. If there is no stabilizing berm, then $k_1= k_2=0$. The properties of the embankment and berm materials are characterized by the shear strength parameters c, ϕ, and the unit weight γ.

The layers of reinforcement are placed at a_1, a_2,…, a_n above the ground surface. When the reinforcement is placed directly on the ground surface, a = 0.

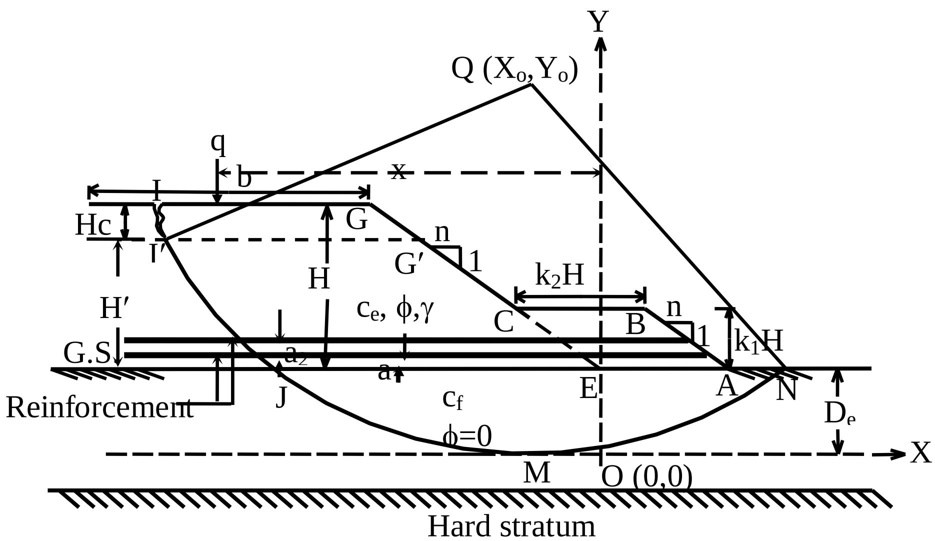

Fig. 1 Reinforced embankment on soft soil with berm and dry tension crack and with applied load at a distance x from the toe of the embankment.

Procedure of rotational stability analysis:

The failure plane is assumed to be a circular arc. Fig. 1 shows an arbitrary failure plane tangential to a limiting tangent at depth D_e. The failure surface encloses the berm. It terminates at the bottom of the tension crack.

The origin of the co-ordinate axes has been taken as the intersection of the limiting tangent and a vertical line passing through the toe E, of the embankment. X_o and Y_o are the co-ordinates of the center of the slip circle.

The factor of safety of the reinforced embankment F is defined as

$$\mathbf{F = \frac{M_{rt}}{M_o}} \qquad \mathbf{(1)}$$

where:

$\mathbf{M_{rf}}$ = moment due to resisting forces in the foundation soil along the slip surface.

$\mathbf{M_o}$ = total overturning moment

Taking into account the effect of the applied loads and using reinforcement in more than one layer will be carried out in deriving the equations for reinforced embankment. These effects were not considered for in the previous studies, and are not taken into account by the basic equation derived by (Kaniraj, 1994). Therefore, this paper will satisfy the derivation of the equations for the location of the critical circle and getting the force of reinforcement required for maintaining the target factor of safety. For the arbitrary failure surface shown in

Figure (1), tangential to the limiting tangent at depth D_e, the factor of safety F is given by equation (1). The total resisting moment in this case consists of three components:

$$\mathbf{M_{rt} = M_{rf} + M_{re} + M_{rr}} \tag{2}$$

where: M_{rr} = moment due to reinforcement force P.

Equations (3) and (4) give M_{rf} and M_{re}, respectively, (Kaniraj, 1994):

$$\mathbf{M_{rf} = 3.06\, c_f\ D_e^{0.53}\ Y_o^{1.47}} \tag{3}$$

where: c_f = equivalent constant undrained cohesion at depth D_e,

D_e = depth of limiting tangent below ground surface.

M_{re} is expressed as (Kaniraj and Abdullah,1994):

$$\mathbf{M_{re} = 1.53\left(c_e + \lambda\gamma H \tan\phi\right) Y_o^{1.47}\left[\left(D_e + H'\right)^{0.53} - D_e^{0.53}\right]} \tag{4}$$

where: c_e = cohesion of embankment soil.

$$\mathbf{H' = H - H_c} \tag{5}$$

H = height of the embankment.

H_c = depth of the tension crack.

γ = unit weight of the embankment soil.

ϕ = angle of shearing resistance of the embankment soil.

λ = averaging coefficient for frictional stress in the embankment

λ is given by Low (1989) as follows:

$$\mathbf{\lambda = 0.19 + \frac{0.02n}{\frac{D_e}{H}} \qquad \text{for } \frac{D_e}{H} \geq 0.5} \tag{6}$$

The total overturning moment (M_o) consists of four components and can be written as:

$$\mathbf{M_o = M_{oe} - M_{ob} + M_{oc} - M_{oq}} \qquad \textbf{(7)}$$

where:

$$\mathbf{M_{oe} = \frac{\gamma H'}{2}\left[X_o\left(nH' - X_o\right) - \frac{\left(nH'\right)^2}{3} + 2Y_o\left(D_e + \frac{H'}{2}\right) - \left(D_e + \frac{H'}{2}\right)^2 - \frac{H'^2}{12}\right]} \qquad \textbf{(8)}$$

$$\mathbf{M_{oc} = \frac{\gamma\left(H - H'\right)}{6}\left[6Y_o\left(D_e + H'\right) - 3\left(D_e + H'\right)^2 - 3X_o{}^2 + 3nX_o\left(H - H'\right) - n^2\left(H^2 + H'^2 + HH'\right)\right]} \qquad \textbf{(9)}$$

Studying the effect of water pressure in a tension crack on the position of the critical circle was found by Spencer (1968) to be rather small, therefore it is neglected in this derivation.

$$\mathbf{M_{ob} = k_1 k_2 \gamma\, H^2\left[\frac{\left(k_2 - nk_2\right)H}{2} + X_o\right]} \qquad \textbf{(10)}$$

M_{oq} (overturning moment due to the applied load) can be written as follows:

$$\mathbf{M_{oq} = q\left(x - X_o\right)} \qquad \textbf{(11)}$$

Kaniraj and Abdulah (1992 b) have not taken this component into account. In this paper, the moment caused by external load q (kN/m) acting at a distance x as shown in Fig. 1 will be accounted for.

If more than one layer of reinforcement is used, then:

$$\mathbf{M_{rr} = P_1\left(Y_o - D_e - a_1\right) + P_2\left(Y_o - D_e - a_2\right) + \ldots + P_n\left(Y_o - D_e - a_n\right)}$$

By more simplification, this expression becomes as below:

$$M_{rr} = (P_1 + P_2 + ... + P_n)Y_o - (P_1 + P_2 + ... + P_n a_2)D_e + ... + (P_1 a_1 + P_2 a_2 + ... + P_n a_n)$$

or:

$$M_{rr} = P_{total}(Y_o - D_e) - P_{total}\left(\frac{P_1 a_1}{P_{total}} + \frac{P_2 a_2}{P_{total}} + ... + \frac{P_n a_n}{P_{total}}\right)$$

Assuming that:

$$P_1 = P_2 = P_n = \frac{P_{total}}{NO}$$

which is similar to the equation derived by (Kaniraj, 1996):

where: NO = number of reinforcment layers.

then:

$$M_{rr} = P_{total}\left(Y_o - D_e - \left(\frac{a_1}{NO}\right) - \left(\frac{a_2}{NO}\right) - ... - \left(\frac{a_n}{NO}\right)\right)$$

or:

$$M_{rr} = P_{total}\left(Y_o - D_e - \sum_{i=1}^{NO}\frac{a_i}{NO}\right) \qquad \textbf{(12)}$$

Substituting for M_{rt} and M_{rr} from eqs. (2) and (12), respectively in eq. (1) gives:

$$\mathbf{F\ M_o = M_{rt}}$$

$$\mathbf{M_{rr} = F\ M_o - M_{rf} - M_{re}}$$

and rearrangement of the equation gives:

$$\mathbf{P_{total}} = \frac{\mathbf{F\ M_o - M_{rt} - M_{re}}}{\left(\mathbf{Y_o - D_e - \sum_{i=1}^{NO} \frac{a_i}{NO}}\right)} \tag{13}$$

The four components of the total overturning moment and their expressions are the same as in the case of unreinforced embankment. The expressions for M_{rr} , M_{rf} , M_{re} and M_o are substituted in eq. (13).

Partial derivatives of eq. (13) with respect to X_o and Y_o are obtained and equated to 0. This gives two equations, the solution of which gives the equations for the coordinates of the center of the critical slip circle as below:

$$\frac{\partial \mathbf{P_{total}}}{\partial \mathbf{X_o}} = \mathbf{0}$$

$$\frac{\gamma_e n H'^2}{2} - \gamma_e H' X_o - k_1 k_2 \gamma_e H^2 - \gamma_e (H - H') X_o + \frac{\gamma_e 3n}{6}(H - H')(H + H') + q = 0$$

$$X_o = \frac{n}{2} H - k_1 k_2 H + \frac{q}{\gamma_e H} \tag{14}$$

and

$$\frac{\partial \mathbf{P_{total}}}{\partial \mathbf{Y_o}} = \mathbf{0}$$

$$\frac{F\gamma_e H'}{2}\left[2\left(D_e + \frac{H'}{2}\right) + \gamma_e (H - H')(D_e + H')\right] - 1.47 H^{1.53} F_2 Y_o^{0.47} = 0$$

or:

$$Y_o^{0.47} = \frac{F\gamma\left[H'\left(D_e + \frac{H'}{2}\right) + (H - H')(D_e + H')\right]}{1.47 H^{1.53} F_2} \tag{15}$$

where:

$$\mathbf{F_2 = \gamma_f A_1 S_f + \gamma_e B_1 S_e} \quad \mathbf{(16)}$$

$$\mathbf{A_1 = 3.06\left(\frac{D_e}{H}\right)^{0.53}} \quad \mathbf{(17)}$$

$$\mathbf{B_1 = 1.53\left[\left(\frac{D_e}{H}+\beta\right)^{0.53} - \left(\frac{D_e}{H}\right)^{0.53}\right]} \quad \mathbf{(18)}$$

$$\mathbf{\beta = \frac{H'}{H}}$$

(19)

$$\mathbf{S_f = \frac{c_f}{\gamma_f H}} \quad \mathbf{(20)}$$

$$\mathbf{S_e = \frac{c_e}{\gamma_e H} + \lambda \tan\phi} \quad \mathbf{(21)}$$

Yo in each case is obtained by solving this equation by trial and error process.

To find P_{max} (The maximum required force), substitute X_o and Y_o in equation (13) and by simplification, the corresponding expression can be obtained as follows:

$$\mathbf{P_{max.} = \frac{F\left(A'_1 - A'_2 + A'_3 - A'_4\right) - H^{1.53} F_2 Y^{1.47}}{\left(Y_o - D_e - \sum_{i=1}^{NO} \frac{a_i}{NO}\right)}} \quad \mathbf{(22)}$$

The values of A1', A2', A3' and A4' are given in equations (23), (24), (25) and (26), respectively. If the value of Pmax is known, a trial and error procedure must be followed to find the factor of safety.

where:

$$A'_1 = \frac{\gamma_e H'}{2}\left[X_o(nH' - X_o) + \left(D_e + \frac{H'}{2}\right)\left(2Y_o - \left(D_e + \frac{H'}{2}\right)\right) - \frac{H'^2}{12}(4n^2 + 1)\right] \qquad (23)$$

$$A'_2 = k_1 k_2 \gamma_e H^2\left[\frac{(k_2 - nk_1)}{2}H + X_o\right]$$

(24)

$$A'_3 = \frac{\gamma_e}{6}(H - H')\left[6Y_o(D_e + H') - 3(D_e + H')^2 - 3X_o{}^2 + 3nX_o(H + H') - n^2\left(H^2 + H'^2 + HH'\right)\right]$$

(25)

$$A'_4 = q(x - X_o) \qquad (26)$$

Kaniraj (1996) gives the working reinforcement force (Pwr) as follows:

$$P_{wr} = \frac{P_{max.}}{F \times NO} \qquad (27)$$

Conditions for the validity of the solution:

For the equations derived in the previous sections to give valid solutions, three assumptions made in the analysis should be satisfied. These are:

a) the center of the slip circle must lie at a level at or above the bottom of the tension crack.

b) The entire berm should lie within the failure plane.

c) The terminal point I' of the failure plane should lie below the crest and not below either of the two side slopes, (see Fig. 1).

Comparison of the proposed solution with Kaniraj (1994) solutions:

To evaluate the accuracy of the proposed equations, the following case is solved by using Kaniraj equations and then resolved using the proposed equations as follows:

H = 6 m, ce=20 kN/m2, ϕ = 0 o , γ = 19.4 kN/m3, n=1.73, D=3 m. The foundation soil has a uniform undrained cohesion of 20 kN/m2 . A comparison of the results is given in Table 1.

The results are also compared with those obtained using Taylour's method (Taylor,1948). A comparison is also made with the results of Bishop's simplified method of slices. This was done through the software program 'PROKON'.

The program 'slopbg' is a part of PROKON . It is a slope stability computer program which uses Bishop's modified method of slices (1955) of analysis for the evaluation of the stability of generalized soil slopes. The ratio of mobilizing and resisting moments on individual slices is used to determine the factor of safety.

Table 1 – A Comparison of the results for case1 (without reinforcement).

Method / Parameter	Proposed Solution	Kaniraj(1994) Solution	Taylor's method (1937 and 1948)	PROKON program
F	1.058	1.054	1.06	1.055
De/H	3/6	3/6	--	3/6

If one layer of reinforcement is used in this embankment, the following results are obtained (Table 2).

Table 2 - A Comparison of the results for case1 (with reinforcement).

Method / Parameter	Proposed Solution	Kaniraj (1994) Solution	PROKON program
F	1.35	1.35	1.35
De/H	3/6	3/6	3/6
Pwr (kN/m)	109	111	103

Illustrative example:

This case is proposed to show the variation of Pmax. , with increasing the factor of safety, and with the increasing of the distance (x) with constant applied load. Fig. 2 shows the geometry of the problem. The properties for this case are given in Table 3.

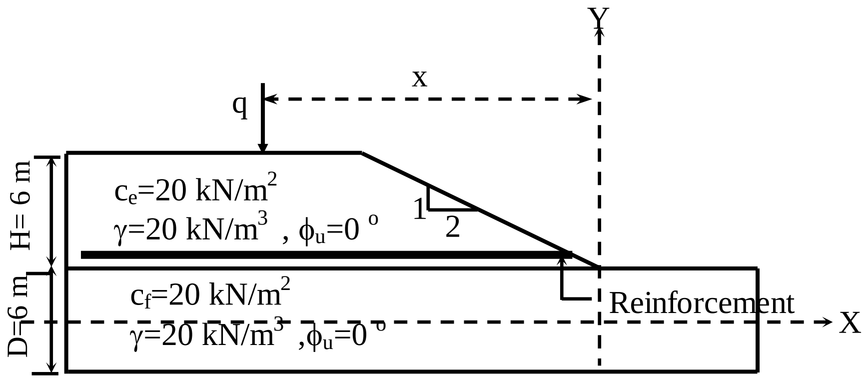

Fig. 2 Geometry of the embankment analyzed in the example.

Table 3 - Material properties for the illustrative example.

	Properties					
Material type	c (kPa)	ϕ^o	γ (kN/m^3)	H (m)	D_e (m)	n
foundation	20	0	20			
embankment	20	0	20	6	6	2

Fig. 3 shows the variation of the value of Pmax., (maximum reinforcement force in kN/m) with the factor of safety under constant applied line load (q=10 kN/m) acting at a constant distance (x=14.5 m). In this part, the value of the factor of safety before applying the reinforcement was (0.96).

Fig. 4 shows the variation of the value of Pmax with the distance (x) under a constant applied line load (q=50 kN/m). The target factor of safety in this case is F=1.35 (when x = 12 m) while (F0) before applying the reinforcement was (0.98).

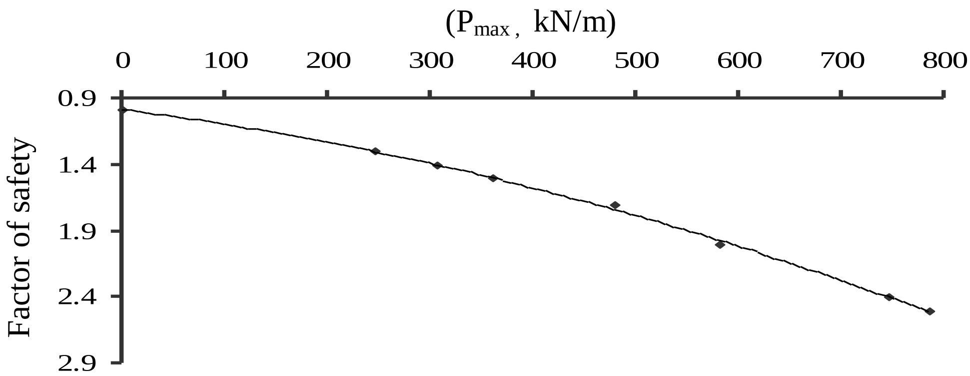

Fig. 3 Variation of the factor of safety, with the maximum reinforcement force, (q=10 kN/m, x=14.5 m).

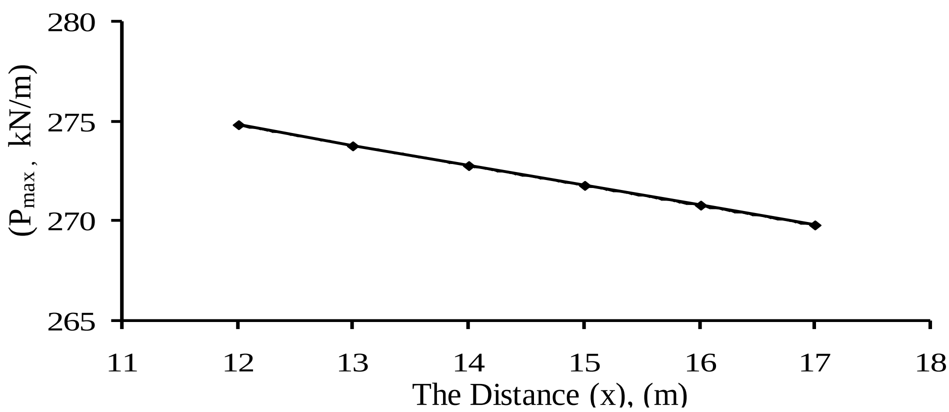

Fig. 4 Effect of the distance of the applied load on the maximum reinforcement force.

Conclusions:

A new solution for the rotational stability analysis of reinforced embankments on soft soils has been presented. The general case of the embankment has a partial height dry tension crack, a berm has been considered. A limit equilibrium method assuming circular slip surface and total stress analysis has been used. Solutions for the location of the critical slip circle and the minimum factor of safety for a given limiting tangent have been presented for reinforced embankments first.

Then the solutions for the location of the critical slip circle and the maximum required reinforcement force for a given limiting tangent have been presented for reinforced embankments. The solutions are expressed in the form of equations. The new approach can take into account the effect of externally applied loads and the reinforcement can be used in more than one layer.

The use of the proposed solution has been verified using example problems. The results obtained using these equations and those obtained using the computer program (PROKON) which incorporates Bishop's simplified method of slices have been compared. The results have also been compared with previous analyses. It was found that the proposed equations presented in this paper give good results.

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NOTATION:

a level of reinforcement above ground surface

b crest width

c_e cohesion of embankment material

c_f equivalent undrained cohesion of foundation soil in depth De

D_e depth of limiting tangent below ground surface

F target factor of safety.

H height of embankment

H_c height of tension crack

$\acute{H}$ uncracked height of embankment = H- Hc

k_1 ratio of berm thickness to embankment height.

k_2 ratio of berm width to embankment height.

M_o total overturning moment

M_{ob} overturning moment due to soil mass in the berm

M_{oc} overturning moment due to soil mass in the embankment in the zone of tension crack

M_{oe} overturning moment due to soil mass in the embankment in the zone of tension crack

M_{re} overturning moment due to soil mass in the embankment below the zone of tension crack

M_{rf} moment due to resisting forces in the foundation soil along the slip surface .

M_{rr} resisting moment due to reinforcement force P

N_1 stability factor for foundation soil.

N_2 stability factor for embankment soil.

NO number of reinforcement layers.

n side slope of embankment (n horizontal to 1 vertical).

P reinforcement force

P_{max} maximum required reinforcement force

P_{wr} working reinforcement force

q applied load . (kN/m)

S_e normalized embankment strength parameter.

S_f normalized foundation strength parameter.

Xí x-coordinate of point í of the slip circle(m)

X_o x-coordinate of the center of the slip circle. (m)

x distance form the origin point(toe of the embankment) to the applied load

Y_o y-coordinate of the center of the slip circle (m)

β ratio of uncracked height to total height of embankment.

γ unit weight of soil .(KN/m^3)

λ averaging coefficient for frictional stress in the embankment.

ϕ angle of internal friction (°)