Military Technical College
Kobry El-Kobbah,
Cairo, Egypt

9th International Conference on Civil and Architecture Engineering ICCAE-9-2012

VALIDATION OF A FINITE DIFFERENCE METHOD FOR FREE SURFACE CALCULATION

HOSSAM ELDIN MOHAMED ELHANAFY *

Military technical college, Cairo, Egypt
**E-Mail: dr_hossam_mohamed@yahoo.com*

ABSTRACT:

This paper describes the validation of a numerical hydraulic model which uses a finite difference method for modelling unsteady free surface water flow over common hydraulic structures. The numerical hydraulic model is based on the Saint Venant equations (SVES) using a staggered finite difference scheme to evaluate the discharge, the water stage, and the cross section area within the domain. While the Modified Method of Characteristics (MMOC) is applied to achieve open boundary downstream and overcome the problem of reflections there. A series of simulations are compared against an existing set of experimental data and other models for the free surface flow over a broad-crested weir. The developed model is promising and capable of simulating different cases of water flow that contain both steady and unsteady flow. This model could be used as a stepping stone for different purposes including parameter identification (Ding et al. 2004), Flood risk assessment (Elhanafy and Copeland 2007) ,uncertainty in the predicted flood (Elhanafy and Copeland 2007) and (Elhanafy et al. 2007).

Keywords: Saint Venant equations (SVES), validation, weirs, CFD, free surfaces, staggered finite difference scheme.

1. INTRODUCTION

The development of hydrological models has gained a lot of momentum in recent years as their ability to describe the spatial and temporal variation of water flow phenomena has been improved.

These models may be used by researchers and engineers or by commercials authorities to produce commercial CFD software. But, before these models could be used, their validation is an important point.

However, now that commercial CFD software has the ability to solve a specified range of engineering problems, the validation material that accompanies them can only ever apply to a subset of these applications.

While academic researchers should compare the CFD simulations with experimental results in order to test the accuracy of the new model. The users of the software assume that the software produces results that can be relied on. The researchers on the other hand, have to validate the software before using them.

Once the hydrological model is validated, it could be used for several purposes such as the prediction of circulation in estuaries for hazardous spill response (Cheng et al. 1993), the prediction of flood wave propagation in rivers (Steinebach 1998), coastal flow modelling (Copeland 1998), and evaluating the sensitivity of the flood to some control variables (Copeland and Elhanafy2006).

2. DESCRIBTION OF THE DEVELOPED MODEL

The Saint Venant equations (SVE$_s$) that take the effect of infiltration rate into consideration form a system of partial differential equations which represents mass and momentum conservation along the channel and include source terms for the bed slope and bed friction. These equations may be written as:

$$\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} - f \cdot b = 0$$

$$\frac{\partial Q}{\partial t} + gA\left(\frac{\partial H}{\partial x} + \frac{\partial z}{\partial x}\right) + \frac{\partial (Qu)}{\partial x} + k\frac{Q|Q|}{A\ R} + \left(\frac{u \cdot f}{2}\right) \cdot b = 0$$

where t is time; x is the horizontal distance along the channel; Q is the discharge; A is the flow cross section area; H is the total water stage; g is the gravitational acceleration; z is the vertical distance between the horizontal datum and the channel bed as function (x,t); S_0 is the bed slope = $-\frac{\partial z}{\partial x}$; k is a friction factor = g/C^2 according to Chezy or = $gn^2/R^{(1/3)}$ according to Manning; and $\frac{\partial (Qu)}{\partial x}$ is the momentum flux term, or convective acceleration; b is the channel bottom width and f is the infiltration rate. The effect of infiltration rate is added to the (SVE^s) using the Green - Ampt model (Green and Ampt, 1911) as follows:

$$f = \frac{dF}{dt} = -K\left(\psi_f + \frac{F}{\theta_s - \theta_i} - H\right)\frac{\theta_s - \theta_i}{F} \qquad (3)$$

Where F is the cumulative depth of infiltration; K is saturated hydraulic conductivity; ψ_f is suction at the wetting front (negative pressure head); θ_i is initial moisture content; θ_s is saturated moisture content ; and H is the depth of ponding.

The estimation of the momentum loss due to seepage (u.f/2) used in the momentum equation (2) follows work by Abiola and Nikaloaos (1998). In simulating an unsteady channel flow during a flood wave event using the Saint Venant equations (SVE^s), equation (1) and equation (2) are subjected to initial and boundary conditions. Initial conditions are Q(x,0) and A(x,0) and the boundary conditions are Q(0,t) and A(L,t) where x = L is the downstream limit of the model domain. Values Q(0,t) comprise the inflow hydrograph and A(L,t) are interpolated from within the domain using the method of characteristics (MOC), (Abbott, 1977) and (French, 1986) after modifying it to suit the case of channel flow over an infiltrating bed as described below, to provide a transparent downstream boundary through which the flood wave can pass without reflection.

4 VERIFICATION TEST CASES

4.1.1 Introduction:

Developing a complete test to check and validate an exact solution for the nonlinear Shallow Water Equations (SWEs) is not possible. It is possible however to develop simple tests to compare the model results with analytical solutions of certain idealized cases. Several tests have been carried out to verify the model from uniform steady flow to non-uniform unsteady flow; we will mention here just the two most important tests.

4.1.2 Validation test 1 – non-uniform unsteady flow:

The main objectives of this test are to assure the following:

- The value of both the discharge (q) and the water depth (H) at the upstream propagate downstream without any change.
- The relationship between q and H follow the analytical solution of the shallow water wave.

The analytical solution of the shallow water wave:

The analytical solution of the shallow water equation in deep water initially, 20 m. with a driving upstream hydrograph following sinusoidal wave concept of amplitude 2.0 m. as illustrated at Figure (3), where; a: is the amplitude of the wave, T: is wave period, t: time, c: wave speed = √ g.H. , and H: is the total water depth is $\eta = a .\{1+ \sin(\theta)\}$ = 2.0 m which lead to H max= 22.0 m and $q = a . \sqrt{(g. H)}. (1+\sin(\Theta)) + q_0$ which lead to q_{max}= 28.01 $m^3/s/m$. and the traveling speed is equal to $\sqrt{(g. H)}$ = 14.69 m/s

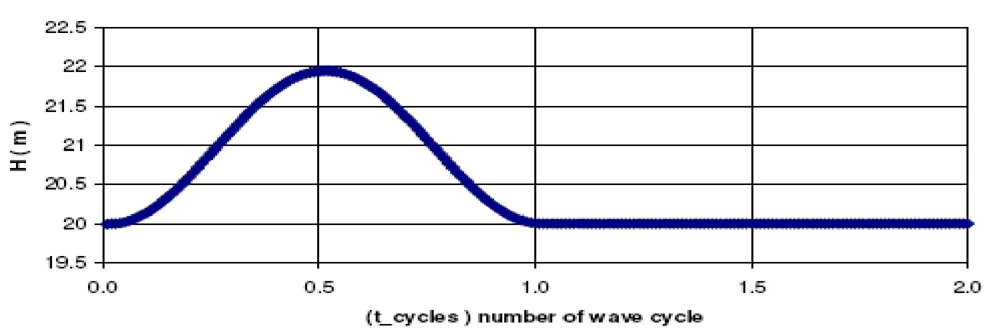

Figure (3) The driving hydrograph shape

The results of the model are a driving upstream boundary hydrograph of peak discharge q = 28.24 m^3/s/m and the calculated upstream boundary hydrograph of peak value, H max = 21.96 m. while the wave speed is 14.74 m/s. so, the first conclusion is that the relationship calculated by the model typically follow the shallow wave equation and although there is discrepancy between the calculated values from the model and the analytical solution but this discrepancy could be interpreted due to the values of distance step = 3.025 Km and time step of 108 s, the second conclusion is that the hydrograph traveled from the upstream boundary to the down stream boundary with a small change in the peak discharge from 28.01 m^3/s/m to 28.24 m^3/s/m and from 21.96 m to 21.94 m for the peak water depth as illustrated at Figure (4) and this acceptable diffusion is duo to the numerical dissipation of the used explicit scheme. The last conclusion is that the wave traveled a distance of 151.26 Km. within 10260 sec. so its speed is 14.74 m/s. while the speed of the wave should equal to √ (g. H) = 14.69 m/s which is nearly the same. So finally, it is clear there is a good agreement between the analytical solution and the developed model and also there is no numerical dissipation.

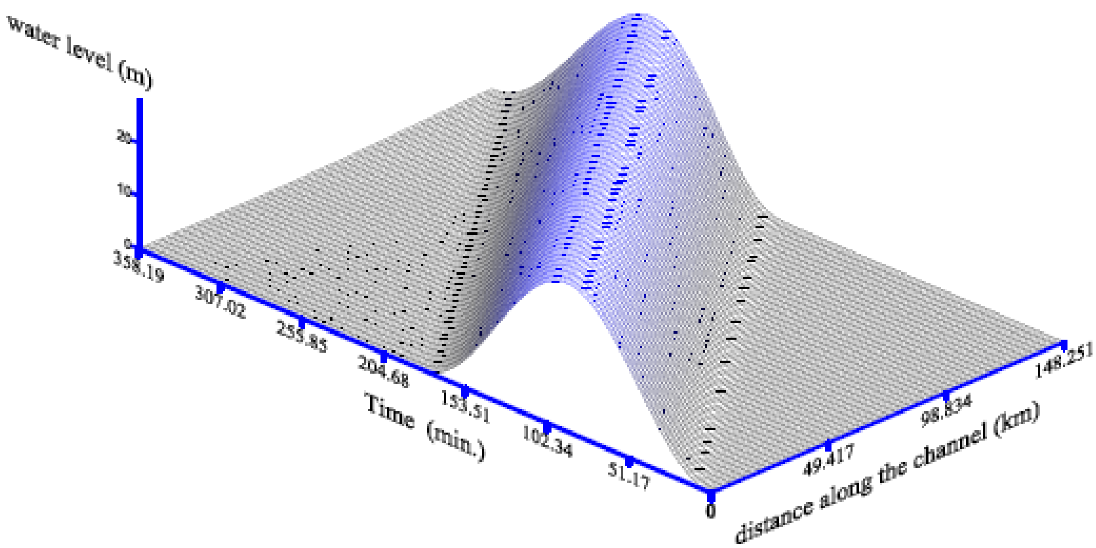

Figure (4) Water depth (H) within the domain

4.1.3 Validation test 2 - Unsteady flow within a sloping channel and rough bed:

There are two main objectives of this test; the first objective is simply to look for the whole channel as a control volume to assure there is no significant losses or accumulation in volume within the simulated domain and the results of this tests will not be compared with the analytical solution only, but will be compared with other model results as well, the second objective is to assure the volumetric conservation principal at different time steps is always valid and no numerical oscillation at the wave front. If we considered the initial water depth is H_i and at the end of the simulation is H_f. While the driving discharge upstream is q_u and downstream is q_d so we could say:

Total volume enters the channel is $\Delta V_1 = \int q_u \, dt - \int q_d \, dt$, while the total volume leaves the channel is $\Delta V_2 = \int H_f \, dx - \int H_i \, dx$, to be in equilibrium, it should be $\Delta V_1 = \Delta V_2$. The model was applied for non-uniform unsteady flow conditions within a slopping channel and rough bed. The initial water depth was chosen H initial = 20.0 m. The result of the flood wave propagation within the domain is presented at Figure (5).

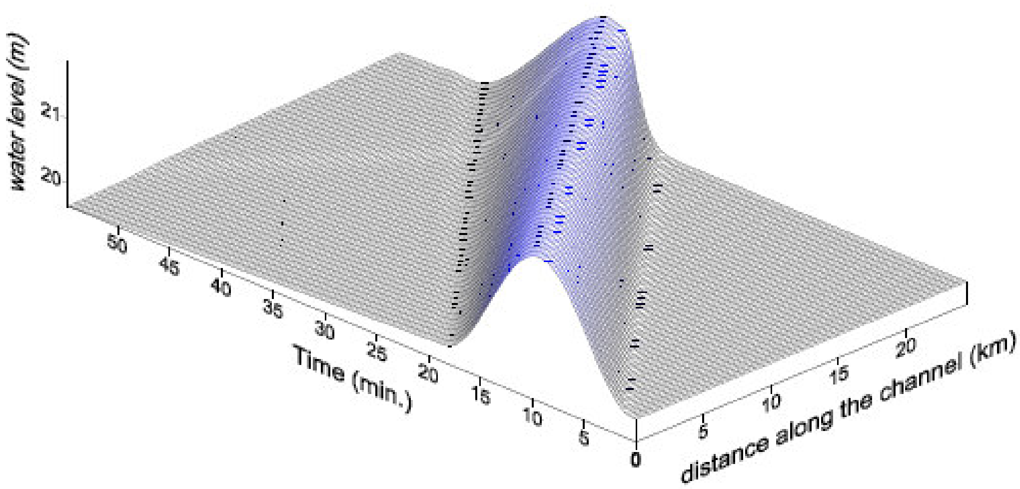

Figure (5) Water depth (H) within the domain

$$\Delta V_1 = \int q_d \, dt - \int q_u \, dt = 38025.58 - 32740.63 = 5284.95 \text{ m}^3\text{/m}$$

$$\Delta V_2 = \int H_i \, dx - \int H_f \, dx = 500000 - 494669.64 = 5330.36 \text{ m}^3\text{/m}$$

So, $\Delta V_2 - \Delta V_1 \cong 45.41$ m3 $\approx$ 0.86 % which is acceptable and it is very small error compared to several previously developed model such as Abiola [15] which was overestimates by 28 %.

4.2 real case Verifications

A case study was applied to the main stream of El-Daba watershed at the north coast of Egypt. The watershed area is about 51km^2, and the main stream runs about 15 km from the outlet station of the watershed. From field measurements (Elhanafy at al, 1999), it is found that the channel cross section is trapezoidal with 14 m. average bottom width and (1:1) side slope as shown in Figure (6). Flow in the channel is simulated for a period of 40 minute during which time a sinusoidal hydrograph shape of duration 13 minute as shown in Figure (7) introduced at the upstream boundary, passes along the channel to represent a flash flood event. The peak discharge is 9.4 m^3.s^{-1}.

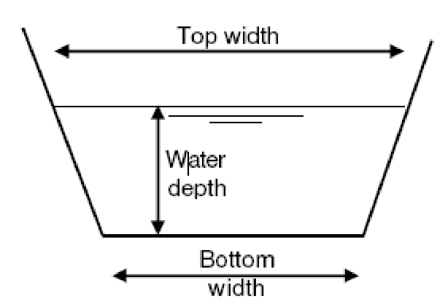

Figure (6) Channel cross section

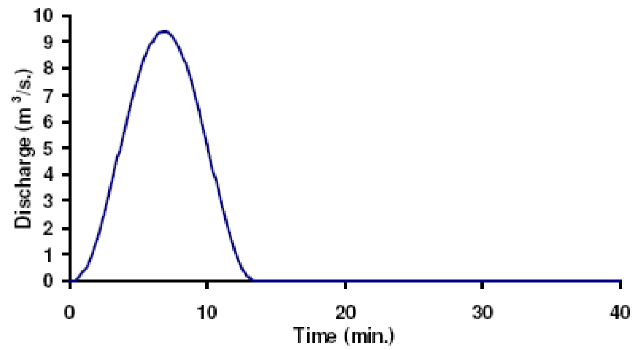

Figure (7) A flash flood hydrograph at the upstream boundary.

5. CONCUSIONS:

The sensitivities expressions relation to the predefined objective function response to upstream driving conditions and to some important control variables which are spatially and temporarily distributed have been derived, and it is clear from Figure (6) that the sensitivity of the flood level at the specified location (x_0) to the upstream discharge follow the hydrograph shape, 15 in other word the sensitivity increases as the discharge increases and decreases as the discharge decreases. While the sensitivity to the bed elevation which is illustrated in Figure (3) explain the effect of the bed elevation from the upstream boundary to the down stream boundary on the threshold water level. Finally, Figure (4) show that the effect of the channel roughness from the upstream boundary till the specified location (x_0) is much more greater than from the specified location (x_0) till the down stream boundary duo to the backwater effect that agree with the basic hydraulic concepts in that any information could propagate upstream only in subcritical flow, which is case studied in this paper. These sensitivities could now be functioned for several purposes, it could be used for parameters identification or it could be used by decision makers to help in prioritizing the most important parameters, in the case studied in this paper as an example, it is clear that the most important control variable is the driving upstream discharge compared to the bed elevation and the channel roughness expressed in Chezy coefficient. or it may be used as a tool to mitigate the flood hazards at certain locations along the channel by identifying the threshold water level not only at x = (x_0) but as function along the studied channel and select the most appropriate location for a certain control action which may be a reservoir or detention dam or a diversion channel. The proper numerical solution and achieving open boundaries for both the forward model and the adjoint problem lead to formulation of an adjoint solution which is consistent with the basic problem. In the near future, the research is to be extended to evaluate both the effect of individual uncertainty in each control variable on the flood event and the global uncertainty from all the control variables on the flood impact.

REFERENCES:

1 - Effect Of Environmental Changes On Drainage Patterns And Urban Planning Along Coastal Regions (Case Study), (Moussa, O.M., Abdelmetaal, N.H., Elmongy, A.E, .and EL-Hanafy, H.M.), pp. 1-17, Third International Conference On Civil & Architecture Engineering , Military Technical College , Kobry Elkobbah, Cairo Egypt, SU9, 1999.

2 - Control of multi-dimensional wave motion in shallow-water, (Sanders, B.F and Katopodes, N.D.), 267-278, Proceedings of the 5th Int. Conf. on Estuarine and Coast. Modeling, M. L. Spaulding, ed., 1998.

3 -Sanders, B.F and Katopodes, N.D., Adjoint sensitivity analysis for shallow-water wave control, Journal of Engineering Mechchanics, ASCE, vol. 126, no. 9, pp 909-919.

4 - Ding, Y. and Wang, S.Y., Optimal control of open-channel flow using adjoint sensitivity analysis, Journal of Hydraulic Engineering, ASCE, vol. 132, no. 11, pp 1215-1228.
5 - Ding, Y., Jia Y., and Wang, S.Y., Identification of Manning's roughness coefficients in shallow water flows, Journal of Hydraulic Engineering, ASCE, vol. 130, no. 6, in press.

6 - Cacuci, D.G., Chapter 3, Sensitivity and Uncertainty Analysis, Volume I, Chapman & Hall CRC., 2003

7 Application of the Adjoint Model in Meteorology, (Navon, I.M. and Zou, X.) Proceedings of the International Conference on Automatic Differentiation of Algorithms: Theory, Implementation and Application, SIAM Publications, A. Griewanek and G. Corliss Eds, SIAM, Philadelphia, PA 1991.

8 - Gejadze, I.Yu and Copeland, G.J.M., Adjoint sensitivity analysis for fluid flow with free surface, Int. J. Numer. Methods in Fluids, vol 47, Issue 8-9,pp 1027-1034.

9 - Computer modelling of channel flow using an inverse method, (Copeland, G.J.M. & El-Hanafy, H.), Proc. 6th Int. Conf. on Civil and Arch. Eng, Cairo, May 2006.

10 - Fletcher, C.J., Chapter 5, Computational Techniques for Fluid Dynamics 1: Fundamental and General Techniques, *Springer-Verlang*., 1991 11 Leonard, B.P., Third-Order Upwinding as a Rational Basis for Computational Fluid Dynamics, *Computational Techniques & Applications: CTAC-83, Edited by Noye, J. And Fletcher, C.,* Elsevier.