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Time-Cost Tradeoff Problems Solutions for Construction Projects Using Integer Linear Programming.

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Abstract:

Time-Cost Optimization (TCO) problem is one of the greatest challenges in construction project planning and control. TCO may be defined as a process to identify suitable construction activit ies for speeding up, and deciding "by how much" so as to attain the best possible savings in both time and cost. In this paper, an Integer Linear Model is developed to obtain solutions of time -cost tradeoff (TCT) problems in construction projects.

The proposed Model collects most related characteristics and constraints presented by different previous works. Also, new factors have been added to the model so as to represents more real TCT problems in life projects. The model is solved by Excel 2007 and verified using the commercial package Primavera (P3) throughout an application example. Good agreement results of model response against different related factors are founded through conducting sensitivity analysis.

Keywords:
Time-Cost Tradeoff, Construction project, Planning

1. Introduction:

Construction Management decisions are made based on developed schedules during the early planning stage of Projects. Decisions could be made by th e expertise using commercial software such as Primavera, Microsoft Project, etc. Sometimes, the output time of the scheduling process does not satisfy the desired conditions (such as the time included in the contract, etc.). Hence, to reduce the project duration, a crashing of some activities durations must be conducted. To perform the activities of a project, planners usually have to decide the construction methods or technologies, operation processes and the associated resources. However, different const ruction methods and related resource combinations for performing an activity create various durations and costs for that particular activity, and therefore, influence the total cost and duration of the project. Hence, planners have to face the decisions of finding the most cost effective way to complete a project within the desirable duration for a project. These decisions are usually made based on the so -called TCT analysis.

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1.1. Time-Cost Tradeoff studies:

Different methods and algorithms were introduced in the literature to deal with TCT problem in construction project management. Mon, et al. [1] proposed a fuzzy PERT/cost that can be applied to a variety of fuzzy distributions of activity durations. They used α -cut method to obtain relationships between project time and cost under different risk levels and different degrees of optimism. Tareghian and Taheri [2] proposed a meta-heuristic solution procedure for the discrete TCQT problem in order to minimize the total cost while maximizing the quality and also meeting a given deadline. Eshtehardian, et al. [3] presented a new approach for the solution of TCT problems in an uncertain environment. Fuzzy numbers were used and Fuzzy sets theory was then explicitly embedded into the optimization procedure. Błaszczyk and Nowak [4] analyzed a project scheduling problem including TCT and considering various resource allocations. Hazir, et al. [5] investigated the budget variant of the discrete TCT problem. This multi -mode project scheduling problem requires assigning modes to the activities of a project so that the total completion time is minimized and the budget and the precedence constraints are satisfied. Anagnostopoulos and Kotsikas [6] evaluated variants of a simulated annealing algorithm which solve the total cost minimization problem in activity networks in the case that discrete time –cost execution modes are allowed on the project activities. Chen and Tsai [7] proposed a novel approach for TCT analysis of a project network in fuzzy environment. The membership function of the fuzzy minimum total crash cost was constructed based on Zadeh's extension principle and fuzzy solutions are provided. M. Rahimi and H. Iranmanesh [8] applied a meta-heuristic algorithm for the discrete TCQT problem and multiple alternatives were considered for the activities of a project.

1.2. Modeling of TCT problems in construction projects using Linear Programming (LP):

The development of LP has been ranked among the most important scientific advances of the mid - 20th century. Its impact has been extraordinary since 1950, and its use has been spreading rapidly. A major proportion of all scientific computation on computers is devoted to the use of LP [9]. Babu and Suresh [10] developed LP models to study the tradeoffs among time, cost, and quality. The model contains traditional activity characteristics and traditional constraints (normal duration, crashed duration, normal direct cost, crashed direct cost, and the relationships among activities are finish to start). Arikan and Gungor [11] presented a practical application of fuzzy goal LP in real life project network with two objectives as minimum completion time and crashing cost wanted to be optimized simultaneously. The membership functions λ1 and λ2 which expressed the total direct cost and total project duration (1=> λ1> = 0; 1=> λ2 >= 0) were summarized in one function (λ). The optimum solution was considered to be the solution produced at λ1= max. No additional activity characteristics or new constraints were included in the model. Sakellaropoulos and Chassiakos [12] developed a solution method considering additional realistic project characteristics such as generalized activity precedence relations (finish to finish [FF], start to start [SS], and finish to start [FS]) and external time constraints for particular activities (an activity can not start or finish after or before a certain time or it must finish at a certain time). The possible existence of lags and leads among activities was considered in the model. Also, penalty and bonuses costs per unit time were applied to the results obtained from the optimization process to get new results. The proposed method is formulated as a linear/integer program and provides the optimal project time –cost curve and the minimum cost schedule. The model considers indirect project costs including general expenses that cannot directly be attributed to particular activities and exist regardless of activity progress (e.g. general office expenses). Indirect costs were typically assumed to be proportional to project duration. Ananya and Chakraborte [13] proposed a method for the minimization of transportation cost as well as time of transportation when the demand, supply and transportation cost per unit of the quantities are fuzzy. The problem is modeled as multi objective LP problem with imprecise parameters. Fuzzy parametric programming has been used to handle impreciseness and the resulting multi objective problem has been solved by goal programming approach.

2. Problem description:

In project planning, one of the most important issues is to achieve the scheduling of project activities. The project duration is an output of the scheduling process. Sometimes, the output duration of the scheduling process does not satisfy the desired conditions (such as the time included in the contract, etc.). Hence, to reduce the project duration, a crashing of some activities durations must be conducted. A simple representation of the possible relationship between the duration of an activity and its direct costs appears in Figure (1).

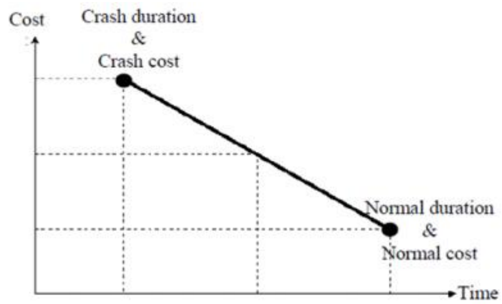


Figure (1): *Illustration of linear time/cost trade-off for an activity*

The linear relationship shown in Figure (1) implies that any intermediate duration could also be chosen (continuous time-cost tradeoff relationship).

The slope of the line connecting the normal point and the crash point is called the cost slope of the activity. The slope of this line can be calculated mathematically by knowing the coordinates of the normal and crash points.

Cost slope = (crash cost – normal cost) / (normal duration – crash duration).

Total project costs include both direct costs and indirect costs. If each activity was scheduled for the duration that resulted in the minimum direct cost in this way, the time to complete the entire project might be too long and substantial penalties associated with the late project completion might be incurred. Thus, planners perform what is called TCT analysis to shorten the project duration. This can be done by selecting some activities on the critical paths to shorten their duration.

As the direct cost for the project equals the sum of the direct costs of its activities, the project direct cost will increase by decreasing its duration. On the other hand, the indirect cost will decrease by decreasing the project duration, as the indirect cost is almost a linear function with the project duration. The decision maker has to choose the appropriate solution (consisting time and corresponding cost) from those plotted on the total cost curve as shown in Figure (2).

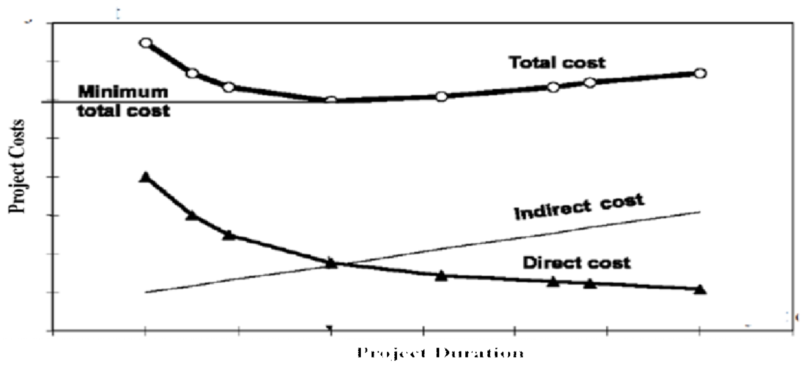


Figure (2)*: Illustration of direct and indirect cost relationships with the project duration.*

To find solutions for the problem, planner can manually solve the problem. However, solving the problem manually can be very difficult and time-consuming, especially if the project consists of a big number of activities. So, an optimization process using a ppropriate technique is needed to find different solutions of the problem. The optimization process is constructed based on a suitable model. The model must consider most conditions and constraints related to this problem. Different conditions and constraints are presented in real projects such as ; different relationships among activities (FS, SS, and FF), external time constraints for particular activities, the possible e xistence of lags and leads among activities, penalty a nd bonuses costs per unit time, indirect cost, total budget, and the presence of holidays.

3. Model (method) development:

Assume the schedule of a project with **n** activities gives a project duration tha t is not satisfied by decision maker. Consequently, the project duration is needed to be crashed, so a study is conducted to get alternatives time-cost data for project activities. To get a new schedule using the original and alternative activities data, a n optimization process must be conducted throughout a suitable model. The proposed model must take into account most factors related to this problem. In this paper an integer linear programming model is suggested to solve this problem. The suggeseted model consists of multiple steps as discussed below.

Step 1:

The following two basic functions presented by [11] for the project network problem are considered:

Min Zc = ∑ **(**Csli***** Tsci**)** $\forall$ i ∈ A (1)

Min Z1 = tfn – ts0 $\forall$ i ∈ A (2)

Where: activity i=1,2,……..,n
A: the set of activities, Zc: total crashed cost for the project, Z1: total completion time for the project, Csli: cost slope for activity i: Csli = Ccmaxi / TCmaxi where:
Ccmaxi: maximum crashed cost for activity i, TCmaxi: maximum possible crashed time for activity i, Tsci**:** solved crashed time for ac tivity I (number of crashed days)
tfn: the finish time of the last activity (n), ts0: the start time of the first activity (0)
These two equations are subjected to the following constraints:

Start time of activity i tsi ≥ 0 $\forall$ i ∈ A

$Tsci \geq 0$ $\forall$ i $\in$ A

$Tsci \leq TCmaxi$ $\forall$ i $\in$ A

$tsj \geq tfi$ $\forall$ i, j $\in$ A

Here: j is the succeeding activity of activity i, tsj: Start time of activity j
Tfi is the finish time for activity i, The constraint tsj ≥ tfi is related to the FS relationships between i and j.

Step 2:

Equation (1) expresses the total crashed cost. To get the total direct cost (Zdc), the sum of normal costs (∑ Cni) of all activities is added to the equation.

Min Zdc = ∑ Cni **+ ∑ (**Csli***** Tsci**)** (3)

Step 3:

Most related factors and characteristics presented in previous works are considered in the proposed model such as; fixed and variable indirect costs, different relationships among activities (FS, SS, and FF), external time constraints for particular activities, the existence of lags and leads among activities, penalty and bonuses costs, and available budget constraint.

Step 4:

The following new factors are used in the model:
- Fixed penalty cost
- Fixed bonuses cost
- Real time factor, which is equal to the ratio between the number of days per week (7days) to the number of working days per week.

Step 5:

Based on the above steps, the new model can be constructed as following:

a) Model functions

Equation (2) calculates the project duration without considering weekly holidays (this means 7 working days per week). If the weekly holidays are presented, then the project duration will increase in a rate equal to:

α = number of days in a week (7days)/number of working days per week.

Therefore, multiplying equation (2) by α will result equation (4) that calculates the real project duration considering the presence of weekly holidays.

Min Z1 = (tfn – ts0)*α (4)

Equation (3) calculates the total direct cost (total crashed cost plus total normal costs). To get the total project completion cost, the following costs must be added to equation (4):

* The indirect cost (indC) [fixed and variable indirect costs].
* The penalty cost (PC) [fixed and variable penalty costs].
* The bonuses cost (BC) [fixed and variable bonuses costs]. Adding these costs to equation (4), we get equation (5).

Min Z2 = ∑ Cni **+ ∑ (**Csli* Tsci**) +** indC +PC –BC $\forall$ i $\in$ A (5)

Here Z2 is the total project completion cost.
indC = indCf +indCv, where indCf is the fixed indirect cost and indCv is the variable indirect cost.
indCv = indd * Z1, where indd is the constant indirect cost per unit time.
PC = PCf + PCv,where PCf is the fixed penalty cost and PCv is variable penalty cost.

PCv = (PT –Z1)*Pd, where PT is the time after which penalty is considered and Pd is the constant penalty cost per unit time.
BC = BCf + BCv, where BCf is the fixed bonuses cost and BCv is the variable penalty cost.
BCv = (BT –Z1)*Bd, where BTis the time before which bonuses is considered and Bd is the constant bonuses cost per unit time.
The finish time of each activity i (tfi) is calculated as:
tfi = tsi + Tni **–** Tsci ;where Tni, tsi, and Tsci are explained in step1.
b) Model constraints
The model equations (4 and 5) are subjected to the following constraints:

tsi ≥ 0 and Tsci ≥ 0 $\forall$ i ∈ A

Tsci ≤ TCmax**i** $\forall$ i ∈ A

For FS relationship, the start time of activity j >= start time of i plus lags (or leads) between j and i (lags have positive values, while leads have negative ones):
tsj ≥ tfi + Lij
For FF relationship, the finish time of activity j ≥ finishes time of i plus lags (or leads) between j and i: tfj ≥ tfi + Lij
For SS relationship, the start time of activity j ≥ start time of i plus lags (or leads) between j and i: tsj ≥ tsi + Lij
Activity i must finish before a certain time (t): tfi ≤ t/α.
Activity i must start before a certain time (t): tsi ≤ t/α.
Activity i must start at a certain time (t): tsi = t/α.
Activity i must finish at a certain time (t): tfi = t/α.
Total completion cost must be equal or smaller than available budget : Z2 ≤ budg.
The start time and the solved crashed time of each activity i must be integer numbers (tsi = integer, Tsci **=** integer). Penalty and bonuses costs are considered in equation (2) with respect to the following conditions:
IF Z1 > PT, THEN: PC = PCf + PCv; ELSE: PC = 0
IF Z1 < BT, THEN: BC = BCf + BCv; ELSE: BC = 0

4. Solution procedure:

1) Project duration (Z1) can be expressed by the membership function (λ1) as presented by [11]: 0 ≤ λ1 ≤ 1: λ1 = (maxZ1 – Z1)/ (maxZ1 – minZ1), Where maxZ1: normal project duration; minZ1: project duration at maximum crashing as shown in Figure 3, Z1: resulted project duration.

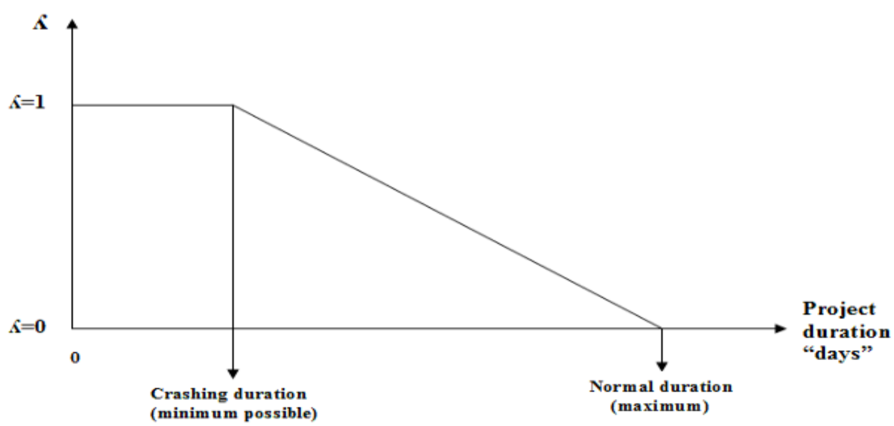


***Figure (3)**: Membership function of project duration*

2) λ1 is divided to 10 intervals ([0 ~ 0.1], [0.1 ~ 0.2], [0.2 ~ 0.3], [0.3 ~ 0.4], [0.4 ~ 0.5], [0.5 ~ 0.6], [0.6 ~ 0.7], [0.7 ~ 0.8], [0.8 ~ 0.9], [0.9 ~ 1]) in addition to λ1= 0, and λ1=1.
3) For each interval of λ1-value, the model is solved to get the minimum total cost (Z2) and corresponding project duration (Z1).
4) Z2-values resulted at each interval are plotted against the corresponding values of Z 1 to get the optimum TCT curve of project activities.
5) The decision-maker can choose the appropriate solution from the resulted time-cost tradeoff curve.

5. An application example:

The data presented by [12] from a highway construction project is used. The project refers to the upgrading of an existing two-lane undivided highway to a four-lane divided motorway with controlled traffic access. In this application, a 100 m road section length is considered for simplicity. The data for the project of 29 activities is shown in Table (1).

***Table (1)**: activities data*

Activity	Normal Time (Tni), day	Crash Time (Tci), day	Normal Cost (lower pound) (Cni), unit	Crash Cost (upper pound) (Cci), unit	Depend.	Lags (+) Or Leads (-)
Service road A						
1-Rock excavation	5	4	2030	2300	0	0
2- Embankment construction	8	6	1020	1510	1(FS)	-3
3- Sub base and base layers	8	6	1700	2090	1(FS) 2(FS)	0 0
4- Asphalt layer	4	3	590	730	3(FS)	0
5- Temporary marking and signing	2	-	90	-	4(SS)	+1
Service road B						
6-Earth and semi-rock excavation	4	3	910	1100	1(FS)	0
7-Embankment construction	2	-	250	-	2(FS) 6(FS)	0 -1
8-Subbase and base layers	7	5	1490	1830	3(FS) 7(FS)	0 0
9-Asphalt layer	4	3	520	750	4(FS) 8(FS)	0 0
10-Temporary marking and signing	2	-	90	-	5(FS) 9(FF)	0 +1
Main road						
11-Traffic diversion	1	-	50	-	5(FS) 10(FF)	0 0
12-Rock excavation	8	6	3260	3710	11(FS)	0
13-Earth and semi-rock excavation—existing pavement	5	3	1140	1720	12(SS)	+2
14- Sub grade stabilization, retaining wall/culvert construction	4	3	300	450	13(SS)	+2

15-Embankment construction	**8**	**5**	**1020**	**1430**	**12(FS)** **14(FS)**	**-4** **-2**
16-Drainage pipe construction	**9**	**6**	**790**	**1180**	**15(FS)**	**-6**
17-Drainage layer	**13**	**11**	**3340**	**4060**	**15(SS)**	**+4**
18-Planting at roadway verges	**9**	**7**	**470**	**830**	**15(FS)**	**+4**
19- El. installations at roadway verges	**6**	**4**	**460**	**810**	**15(FS)**	**0**
20- Ditches	**6**	**5**	**1280**	**1430**	**17(SS)**	**+3**
21- Sub base layer	**14**	**10**	**1090**	**1560**	**20(SS)**	**+2**
22- Base layer	**14**	**9**	**900**	**1400**	**21(SS)**	**+2**
23- Median island (New Jersey)	**14**	**11**	**2220**	**2690**	**22(FS)**	**-9**
24- Elect. installation in median island	**3**	**-**	**230**	**-**	**23(SS)**	**+6**
25- Asphalt layer #1	**6**	**4**	**1590**	**1990**	**23(FS)**	**-4**
26- Asphalt layer #2	**10**	**8**	**2630**	**3240**	**25(SS)**	**+4**
27- Friction course overlay	**8**	**6**	**2060**	**2660**	**26(FS)**	**0**
28 -Final marking and signing	**10**	**8**	**320**	**610**	**27(FS)**	**-3**
29- Traffic restoration	**1**	**-**	**50**	**-**	**28(FS)**	**0**

An external constraint is set for the completion time of the service road s. In particular, the latest finish time of activity 11 is 23 days after the beginning of the project. The variable indirect project cost is 150 units per day. Further, a variable penalty at a rate of 200 units per day of delay applies after the 80th day while a variable bonus of 100 units per day is given for project completion before the 80th day.

In this case study, fixed indirect, penalty and bonuses costs were not considered (this means that indCf =0, PCf = 0, and BCf = 0). The holiday days were not taking into account (7 working days per week), so in our model this can be expressed by giving a value equal to (1) to the real time factor (α). Available project budget is not presented, so this constraint will not be considered in the solution. The project network is shown in Figure (4).

To get the optimum solution for project problem, the membership function is expressed as: $\lambda 1 = (maxZ1 – Z1)/ (maxZ1 – minZ1)$.

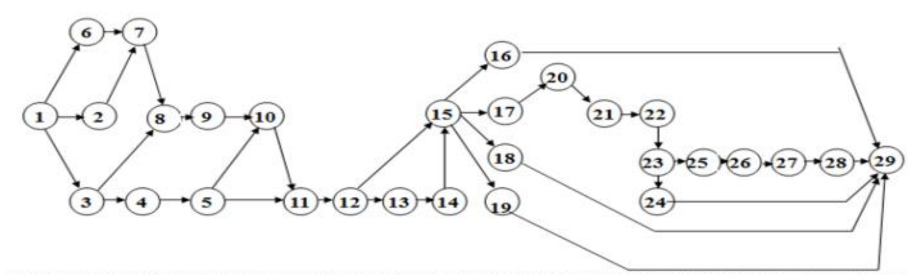


Figure (4): *project network*

-Model running at minimum possible crashing (this means at mi nimum direct cost), we get project duration = maxZ1 = 85 days. Model running at minZ1, we get project duration = minZ1 = 70 days. So, membership function can be presented as:

λ1 = (85 – Z1)/ (85 – 70).

-Model running for all λ1-intervals at minZ2, we calculate Z2-values and corresponding Z1-values. The model solutions are presented in Table (2) and plotted in Figure (5).

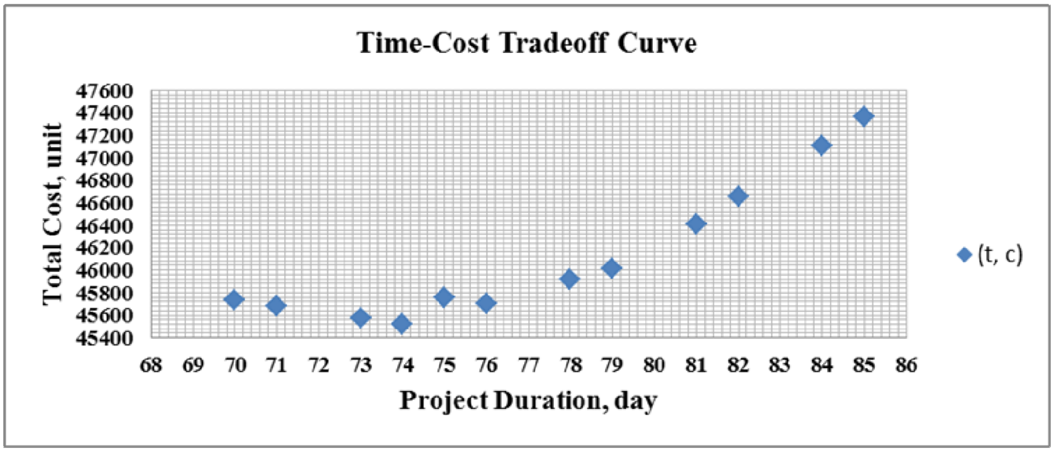


***Figure (5)**: Optimum Time-Cost Tradeoff Curve for original case study*

It can be seen that minZ2 to complete the project can be achieved by Z1 equal to 74 days. It is noted that, all solutions with Z1 ≥ 75 are dominated by the solution (Z1= 74, Z2 = 45521).

***Table (2):** Time-cost solutions*

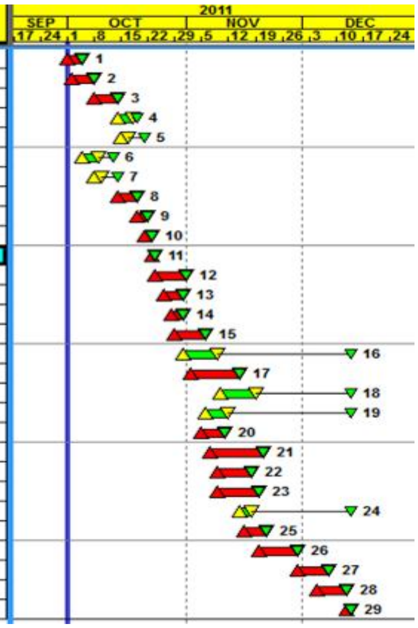
λ1-interval interval	Z1, days	λ1-value	Direct Cost	Indirect Cost	Bonuses Cost	Penalty Cost	Z2, unit
0	85	0	33610	12750	0	1000	47360
0-0.1	84	0.06667	33710	12600	0	800	47110
0.1-0.2	82	0.2	33960	12300	0	400	46660
0.2-0.3	81	0.26667	34055	12150	0	200	46405
0.3-0.4	79	0.4	34260	11850	100	0	46010
0.4-0.5	78	0.46667	34417	11700	200	0	45917
0.5-0.6	76	0.6	34707	11400	400	0	45707
0.6-0.7	75	0.66667	35007	11250	500	0	45757
0.7-0.8	74	0.73333	35021	11100	600	0	45521
0.8-0.9	73	0.8	35321	10950	700	0	45571
0.9-1	71	0.93333	35931	10650	900	0	45681
1	70	1	36231	10500	1000	0	45731

Of course, this will not be a reason to exclude these solutions. The project team might not be able to execute the activities with the applied crashing (for the first four solutions). Therefore, the presence of other solutions is very important.

To verify the schedule of project activities, we use the new resulted activities durations (for solu tion Z1=74 days and Z2 = 45521) as an input data. This can be done manually or using commercial software such as primavera project planner (P3). The resulted schedule is shown in Figure (6).

Figure (6)*: Project schedule at solution Z1=74 days and Z2=45521 units.*

Activity ID	Activity Description	Orig Dur	Rem Dur	%	Early Start	Early Finish
10	1	4	4	0	01OCT11	04OCT11
20	2	6	6	0	02OCT11	07OCT11
30	3	6	6	0	08OCT11	13OCT11
40	4	4	4	0	14OCT11	17OCT11
50	5	2	2	0	15OCT11	16OCT11
60	6	4	4	0	05OCT11	08OCT11
70	7	2	2	0	08OCT11	09OCT11
80	8	5	5	0	14OCT11	18OCT11
90	9	3	3	0	19OCT11	21OCT11
100	10	2	2	0	21OCT11	22OCT11
110	11	1	1	0	23OCT11	23OCT11*
120	12	8	8	0	24OCT11	31OCT11
130	13	5	5	0	26OCT11	30OCT11
140	14	3	3	0	28OCT11	30OCT11
150	15	8	8	0	29OCT11	05NOV11
160	16	9	9	0	31OCT11	08NOV11
170	17	13	13	0	02NOV11	14NOV11
180	18	9	9	0	10NOV11	18NOV11
190	19	6	6	0	06NOV11	11NOV11
200	20	6	6	0	05NOV11	10NOV11
210	21	14	14	0	07NOV11	20NOV11
220	22	9	9	0	09NOV11	17NOV11
230	23	11	11	0	09NOV11	19NOV11
240	24	3	3	0	15NOV11	17NOV11
250	25	6	6	0	16NOV11	21NOV11
260	26	10	10	0	20NOV11	29NOV11
270	27	8	8	0	30NOV11	07DEC11
280	28	8	8	0	05DEC11	12DEC11
290	29	1	1	0	13DEC11	13DEC11



It can be seen that project starts at 1/10/2011 and finishes at 13/12/2011 (74 days). Also activity 11 finishes at 23/10/2011 (23 days after project start).

6. Sensitivity analysis:

In this section, the sensitivity of the proposed model against different factors is studied. This is done by an illustrative sensitivity analysis for the application example.

6.1. Available budget:

Assume a constraint budget of 46000 units is considered for the project. The resulted TCT curve is shown in Figure (7).

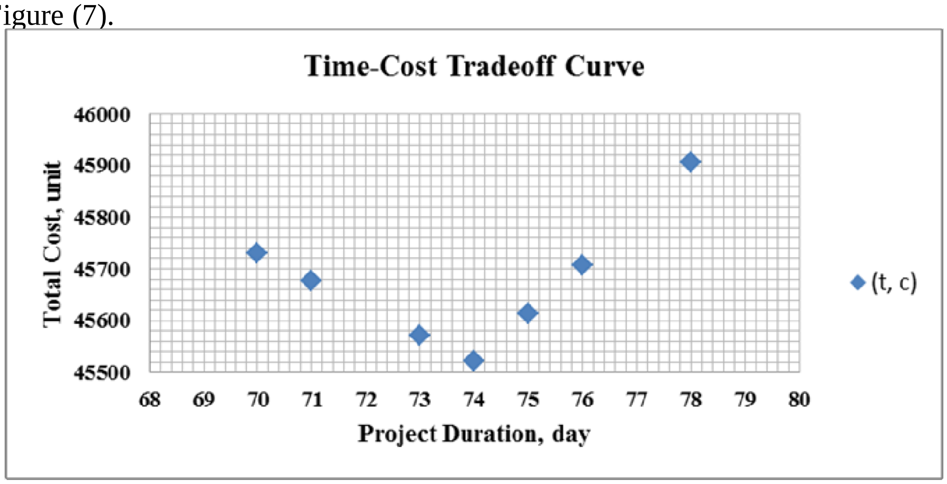


Figure (7)*: Optimum TCT curve at case where available budget<= 46000 units.*

It can be shown that all solutions with Z2 > 46000 are excluded from the original TCT curve.

6.2. Generalized activities characteristics:

Considering activity 15 cannot start before 35 days from the project start, then the TCT curve at this case is presented in Figure (8).

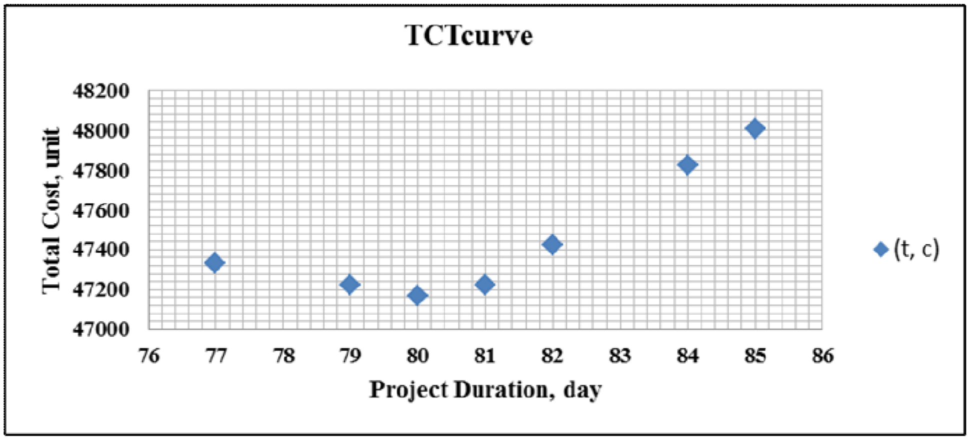


Figure (8): *Optimum TCT curve at case where activity 15 cannot start before 35 days*

It can be seen that Z1-values increased due to the increase of start time of activity 15. The model solution (Z1=80 days, Z2 = 47171 units) is solved by P3 as shown in Figure (9).

Activity ID	Activity Description	Orig Dur	Rem Dur	%	Early Start	Early Finish
10	1	4	4	0	01OCT11	04OCT11
20	2	6	6	0	02OCT11	07OCT11
30	3	6	6	0	08OCT11	13OCT11
40	4	4	4	0	14OCT11	17OCT11
50	5	2	2	0	15OCT11	16OCT11
60	6	4	4	0	05OCT11	08OCT11
70	7	2	2	0	08OCT11	09OCT11
80	8	5	5	0	14OCT11	18OCT11
90	9	3	3	0	19OCT11	21OCT11
100	10	2	2	0	21OCT11	22OCT11
110	11	1	1	0	23OCT11	23OCT11*
120	12	8	8	0	24OCT11	31OCT11
130	13	5	5	0	26OCT11	30OCT11
140	14	4	4	0	28OCT11	31OCT11
150	15	8	8	0	05NOV11*	12NOV11
160	16	9	9	0	07NOV11	15NOV11
170	17	13	13	0	09NOV11	21NOV11
180	18	9	9	0	17NOV11	25NOV11
190	19	6	6	0	13NOV11	18NOV11
200	20	6	6	0	12NOV11	17NOV11
210	21	14	14	0	14NOV11	27NOV11
220	22	9	9	0	16NOV11	24NOV11
230	23	11	11	0	16NOV11	26NOV11
240	24	3	3	0	22NOV11	24NOV11
250	25	6	6	0	23NOV11	28NOV11
260	26	10	10	0	27NOV11	06DEC11
270	27	7	7	0	07DEC11	13DEC11
280	28	8	8	0	11DEC11	18DEC11
290	29	1	1	0	19DEC11	19DEC11

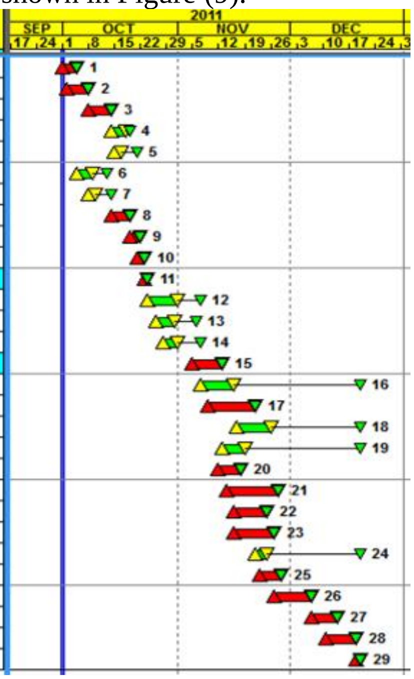


Figure (9)*: project schedule at solution Z1=80 days and Z2=47171 units.*

It is noticed that activity 15 starts at 5/11/2011 (this means 35 days after project starts) and project finish at 19/12/2011 (80 days after project start).

6.3. Fixed bonuses cost:

In addition to the variable bonuses cost in the original case , a fixed bonuses cost (1000 units) for a project completion duration earlier than 80 days is considered. The resulted TCT curve is presented in Figure (10).
It is noted that a decreasing of total cost has been occurred for all solutions with Z 1 < 80 days due to the adding of fixed bonuses cost.

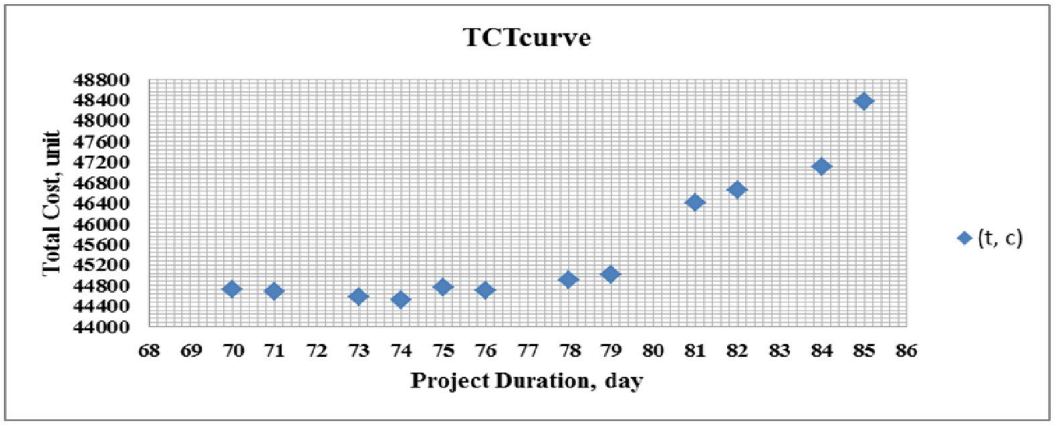


***Figure (10)**: Optimum TCT curve considering1000 units of fixed bonuses cost*

6.4. Fixed penalty cost:

In addition to the variable penalty cost in the original case , a fixed penalty cost (1000 units) for project completion durations later than 80 days is considered. The resulted TCT curve is presented in Figure (11).

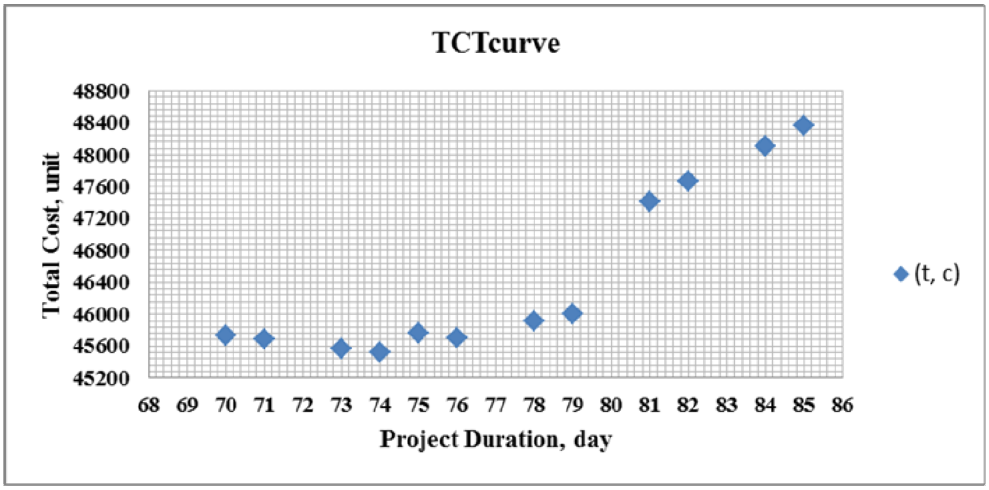


***Figure (11)**: Optimum TCT curve considering 1000 units of fixed penalty cost*

It can be seen that an increasing of total cost has been occurred for all solutions with Z 1 > 80 days due to the added fixed penalty cost.

6.5. Fixed indirect cost:

In addition to the variable indirect cost in the original case, a fixed indirect cost equal to 1000 units is considered. The resulted TCT curve is presented in Figure (12). It is observed that an increasing of total cost has been occurred for all solutions.

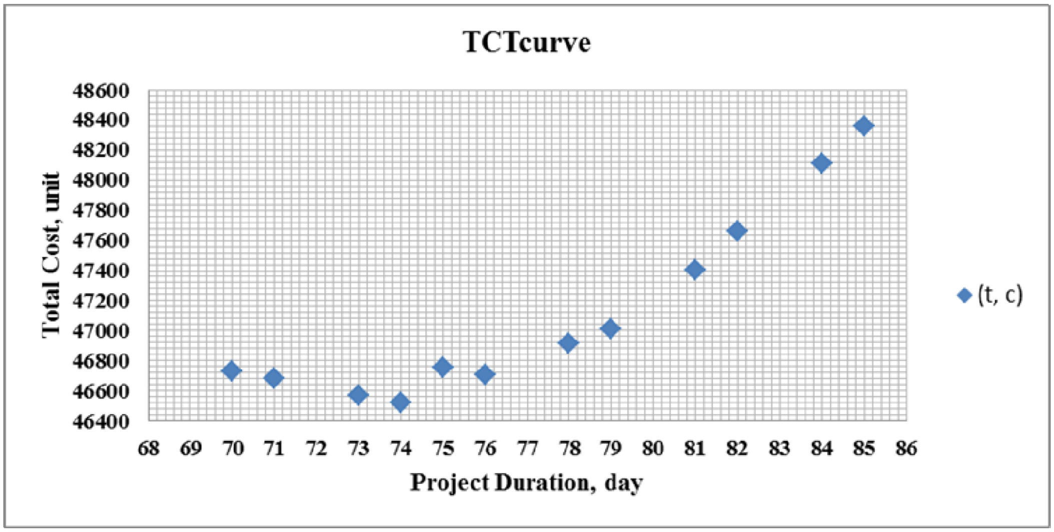


***Figure (12)**: Optimum TCT curve considering fixed indirect cost (1000 units)*

6.6. Real time factor:

Considering 5 working days per week, the real time factor (α) = 7/5 =1.4. TCT curve for this case is drawn as follows:

1- Membership function will take the following new boundaries:

λ1 = (85*1.4 – Z1)/ (85 – 70)*1.4 = (119 – Z1)/ (119 – 98).

2- The constraint related to activity 11 (activity 11 finish at 23 days after project start) is given for a case of 7 working days per week. In this case the equivalent finish time of activity 11 equal to 23*1.4 ≈ 32 days. The resulted TCT curve is plotted in Figure (13).

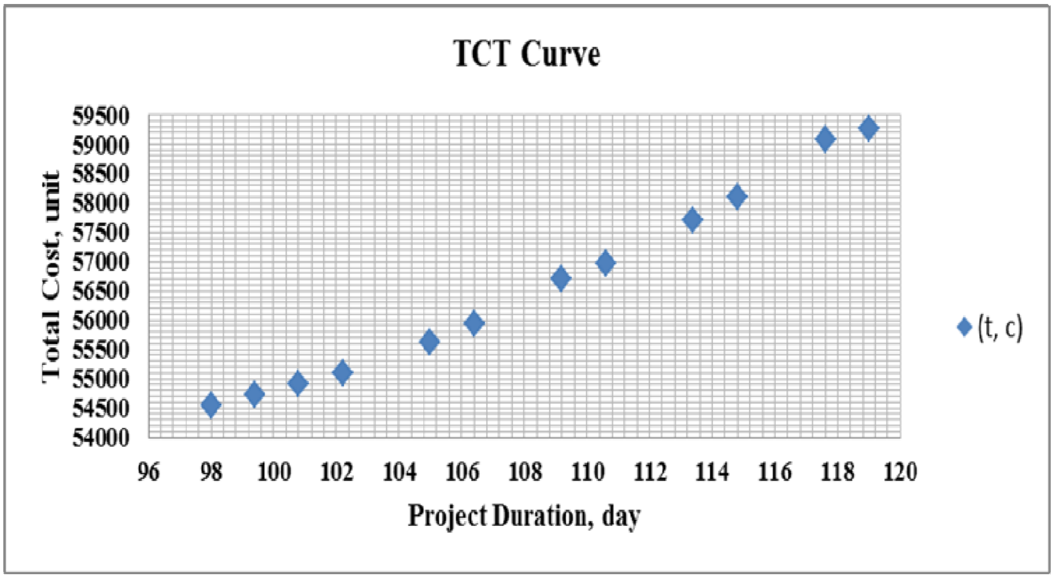


***Figure (13)**: Optimum TCT curve considering 5-working days per week (α = 7/5 = 1.4)*

It is clear that a valuable increasing in the total cost has been occurred due to the increasing of indirect and penalty costs. The existence of 2 holiday days per week caused an increasing in project duration (Z1). Indirect and penalty costs increased proportionally with respect to the increase of Z1. To verify this case, P3 is used to schedule project activities at the solution Z 1 = 98 days and Z2 = 54531 units. The schedule is shown in Figure (14).
It is noticed that project starts at 1/10/2011 and finishes at 7/1/2012. (99 days), however, the model results duration is 98 days. The one-day difference is related to the reason that, P3 is affected by the chosen of holiday days, project start day, and the resulted finish day, while the model cannot consider these factors.

Activity ID	Activity Description	Orig Dur	Rem Dur	%	Early Start	Early Finish
10	1	4	4	0	01OCT11	04OCT11
20	2	6	6	0	02OCT11	09OCT11
30	3	6	6	0	10OCT11	17OCT11
40	4	4	4	0	18OCT11	23OCT11
50	5	2	2	0	19OCT11	22OCT11
60	6	4	4	0	05OCT11	10OCT11
70	7	2	2	0	10OCT11	11OCT11
80	8	5	5	0	18OCT11	24OCT11
90	9	3	3	0	25OCT11	29OCT11
100	10	2	2	0	29OCT11	30OCT11
110	11	1	1	0	01NOV11	01NOV11*
120	12	8	8	0	02NOV11	13NOV11
130	13	5	5	0	06NOV11	12NOV11
140	14	3	3	0	08NOV11	12NOV11
150	15	8	8	0	09NOV11	20NOV11
160	16	9	9	0	13NOV11	23NOV11
170	17	13	13	0	15NOV11	03DEC11
180	18	9	9	0	27NOV11	07DEC11
190	19	6	6	0	21NOV11	28NOV11
200	20	6	6	0	20NOV11	27NOV11
210	21	14	14	0	22NOV11	11DEC11
220	22	9	9	0	26NOV11	06DEC11
230	23	11	11	0	26NOV11	10DEC11
240	24	3	3	0	04DEC11	06DEC11
250	25	6	6	0	05DEC11	12DEC11
260	26	8	8	0	11DEC11	20DEC11
270	27	6	6	0	21DEC11	28DEC11
280	28	8	8	0	26DEC11	04JAN12
290	29	1	1	0	07JAN12	07JAN12

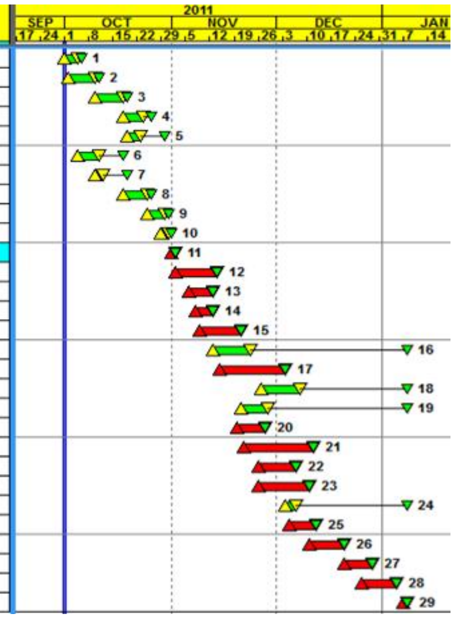


***Figure (14)**: Project schedule for 5-working days per week (Z1 = 98 days, Z2 = 54531units)*

7. Conclusion:

In this work, an Integer Linear model combining most factors and characteristics presented in previous works and related to TCT problems was proposed. Also, other factors such as fixed indirect, fixed penalty, and fixed bonuses costs were introduced in the model. for best real representation of life projects, the weekly holiday days were considered in the model by introducing real time factor (α) = 7/ (number of working days per week). A procedure method to get optimum time-cost solutions was presented. The optimum time-cost solutions were plotted in so-called TCT curve to enable decision maker to choose the appropriate solution for the problem.
A sensitivity analysis was conducted to test the model reaction against different affected factors. The sensitivity analysis showed good reaction of the model to the different solved cases. The new resulted activities durations (after crashing) for different solutions were scheduled using primavera project planner (P3). The aim of this process was to ensure that model scheduled project activities at minimum durations. The comparison showed that results were fitting well.

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