Growth and Instability of Long Cracks in Non-redundant and Redundant Structures

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**ABSTRACT**

The propagation of very long fatigue cracks in a redundant and non-redundant stiffened-panel structure is assessed. In ship structures, cracks are known to propagate to a length greater than 24m without sudden fracture or instability. To investigate this phenomenon, large stiffened single panels and cellular box beams were fatigue tested and the crack length versus number of cycles monitored. The panels failed by ductile tearing at service temperatures due to net section plastic collapse. In addition, probabilistic numerical finite element models were developed and the solution for the stress intensity factor (K) obtained and used with the Paris Law to characterize the crack propagation rate. The results can be used to specify inspection and maintenance intervals to avoid complete system failure.

**Keywords:** Ship, Long cracks, Fatigue, Weld, Residual Stresses, Stress intensity factor

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**1.0 Introduction**

Fatigue cracking in modern ships is a serviceability problem rather than a structural integrity problem. A large tanker may have hundreds or even thousands of fatigue cracks discovered during inspection. Fatigue cracks could grow to be as large as 24 meters as shown in Figure 1. These long cracks remain surprisingly stable because of minimum notch-toughness levels specified for ship steel. However, even if brittle fracture does not occur, at some point a net-section plastic collapse of the structure will occur, leading to a failure of the entire ship (Dexter and Mahmoud, 2004). Many experiments verified the stability of long crack propagation in redundant structures (Dexter and Gentilcore, 1997); (Nussbaumer, Fisher, and Dexter, 1999); and (Dexter and Pilarski, 2000). Experiments on single non-redundant stiffened panels also verified the stability of the propagation of long cracks (Dexter and Mahmoud, 2004). The experiments showed a substantial decrease in the crack growth rate between stiffeners, which is attributed to compressive residual stress between stiffeners. Experiments had also shown that stable growth of long fatigue cracks can be reasonably well predicted. However, it is not clear as to how to predict the growth rate of long fatigue cracks in complex welded structures (Dexter, Pilarski, and...
This paper describes the experimental research conducted to develop models to predict the growth rate of these long fatigue cracks in redundant stiffened box-sections and non-redundant single stiffened panels. An analytical model is described that estimates the stress-intensity factor based on superposition of linear-elastic fracture mechanics solutions.

Figure 1 - Cracked deck in the tanker Castor  
(Courtesy of www.imo.org/newsroom/mainframe.asp?topic_id=72)

2.0 Crack Propagation Models
Fatigue crack growth is governed by the range in stress-intensity factor, $\Delta K$ (Paris and Edorgan, 1963). Experimental $da/dN$ verses $\Delta K$ data typically exhibit a sigmoid shape when plotted on a log-log scale. The Paris Law is fit to the linear portion of the $da/dN$ versus $\Delta K$ plot (on a log-log scale), which lies above $\Delta K_{th}$. The $\Delta K$ threshold, $\Delta K_{th}$, is a value below which cracks will not propagate. For structural steel, $\Delta K_{th}$ can be taken between 2.7 and 3 MPa-m$^{1/2}$. At relatively high $\Delta K$ levels, the crack growth rate accelerates and is accompanied by ductile tearing or increments of brittle fracture in each cycle. The Paris Law is expressed as:
\[ \frac{da}{dN} = C \cdot (\Delta K)^m \]  

Equation (1)

where:

\( a \) = half the crack length \\
\( N \) = number of cycles \\
\( C \) = an experimentally determined coefficient \\
\( \Delta K \) = stress intensity factor range \\
\( m \) = material constant

The value of \( m \), the exponent in the Paris Law, is typically taken as 3.0 for steel in air. Many studies were performed to establish an upper bound value of the coefficient \( C \) for a variety of ferritic steels. Therefore, most reported values of the coefficient \( C \) are intended to represent a conservative upper bound to the data. Barsom and Rolfe (1987) established an upper bound value of \( C \), which is equal to \( 6.8 \times 10^{-12} \) for units of MPa and meters. The British Standards Institute BS 7910 (1999) recommends an upper bound of \( 16.5 \times 10^{-12} \) for \( C \). However, this value seems to be excessively conservative. In the PD 6493 document, which is a previous document of BS 7910, a more reasonable upper bound of \( 9.5 \times 10^{-12} \) was recommended for \( C \). Dexter and Pilarski (2000) and Dexter and Mahmoud (2004) also concluded that a \( C \) value of \( 9.5 \times 10^{-12} \) is an appropriate value to use for representing the actual propagation rate in the plate, keeping in mind that the proper value of compressive residual stresses is also used in the analysis. Fisher, et al. (1993) performed a study on HSLA-80 steel, which showed that the upper bound value of \( C \) for high-load-ratio data was \( 9.0 \times 10^{-12} \).

3.0 Stiffener Effect

Much of the research conducted on crack propagation in stiffened panels investigated crack growth in stiffened aircraft aluminum sheets. Fatigue crack growth rates in aluminum panels with both riveted and integral stiffeners were studied by Poe (Poe, 1969, 1971). Poe proposed a numerical solution for \( K \) in a cracked panel with uniformly spaced riveted stiffeners and introduced a parameter called “percent stiffening”, to account for stiffener restraint, which compares the area of the stiffeners to the total area of the stiffeners and the plate. Through Poe’s experimental work, the beneficial ef
restraint and load shedding was shown. Load shedding takes place when load is transferred to the stiffener as the load originally carried by the panel and the stiffeners is transferred to the stiffeners. The decrease in the K solution as the crack approaches the first stiffener is due to the effect of stiffener restraint (Figure 2).

In panels with welded or integral stiffeners the crack may propagate directly into the stiffeners and completely sever it. Poe (1971) proposed a solution that assumes that the stiffener is suddenly and completely severed once the crack has entered it. The sudden loss of the stiffener would then cause a sudden jump in the K solution as shown by the dashed lines in Figure 2. However, realizing that the crack grew in the stiffener with the same rate it grew past the stiffener, Poe proposed a linear increase in the stress intensity factor between the solution for an intact stiffener and a completely severed stiffener. The linear interpolation was between the point on the panel where the stiffener is located and the point at a distance beyond the stiffener, which is equal to the height of the stiffener web (Figure 2).

![Diagram of K solution for a panel with integral stiffeners](image)

**Figure 2 – K solution for a panel with integral stiffeners (Poe, 1971)**

The effect of severed stiffeners on the solution of K is well represented by symmetrically located pair of forces acting on the crack face and is given by the Greene’s function as:
Equation (2)

\[ K = \frac{2F}{t_{pl} \sqrt{\pi a}} \frac{a}{\sqrt{a^2 - s^2}} \]

where;

F is the magnitude of the force in the stiffener (stress times area of the stiffener);

a is the half crack length;

s is the distance of the stiffeners from the centerline of the crack; and,

t_{pl} is the thickness of the shell plate.

The Green’s function used for modeling the effect of severed stiffeners was also used to analytically model the effect of residual stresses (Dexter and Pilarski, 2000); (Dexter and Mahmoud, 2004). Assuming no redistribution during cracking, the residual stress intensity factor (K\text{r}) was calculated using equation 2 by treating the stress on the face of the crack as a splitting force. The total solution for K\text{r} was found by integrating the Greene’s function along the crack face. An example of the resulting K\text{r} versus half the crack length is shown in Figure 3.

![Image of graph showing Kr versus half crack length](image)

Figure 3: Stress intensity factor (Kr) resulting from the residual stress

### 4.0 Crack Closure

Crack closure is a phenomenon, which has the tendency to keep the crack closed under limited amount of applied tension. A portion of the tensile loading may not contribute
new crack growth, but rather to open the crack. Generally, many factors contribute to a closure of a crack such as plasticity effect, corrosion products, asperity mismatch, and compressive residual stresses effect. The effective tensile loading needed to open a crack was defined by (Elber, 1970) as:

\[ \Delta \sigma_{\text{eff}} = \sigma_{\text{max}} - \sigma_{\text{op}} \]  
Equation (3)

and

\[ \Delta K_{\text{eff}} = K_{\text{max}} - K_{\text{op}} \]  
Equation (4)

where \( \sigma_{\text{op}} \) is the crack opening stress and \( K_{\text{op}} \) is defined as the amount of stress intensity factor necessary for the crack front to open. At high load ratios the crack exhibits no closure effect, where \( \Delta K_{\text{eff}} \) is equal to \( \Delta K \).

Plasticity effect is generated by the plastic deformation at the crack tip, which induces closure effect on the crack as well as the resistance of plastic strain reversal ahead of the crack tip. In the case of long fatigue cracks, plasticity effect is minimal and can be ignored.

Compressive residual stress from welding or other fabrication processes is considered a major contributor to crack closure. The effect of tensile residual stress is essentially the same as the effect of high load ratio; it can eliminate any effect of crack closure. In the presence of tensile residual stress from welding, even the compressive part of the applied stress range is effective.

Many attempts have been made to accurately measure the magnitude of compressive stresses in welded structures. A comparison study between various techniques used for measuring residual stresses showed that the sectioning method yields the most accurate results with standard deviation of \( \pm 10 \) MPa when measured in a normal case (J. Lu, 1996).

Dexter and Pilarski (2000) measured the magnitude of residual stresses in two welded stiffened box-sections. As shown in Figure 4, there is a great deal of scatter in the residual stress values measured in two difference specimens. Furthermore, equilibrium requires offsetting areas of tensile and compressive stress to balance in the specimen. A worse-case scenario was used for the numerical and analytical models developed.
Dexter and Mahmoud (2004) measured residual stresses in three single welded stiffened panels (Figure 5). As expected, there was scatter in the measured values of residual stresses. The measured values in all three specimens varied between 50 and 100 MPa. For equilibrium to take place, assuming that the tensile residual stress is 350 MPa at the weld, a constant compressive residual stress value of approximately 75 MPa must be present in the plate. This assumption was verified in the F.E. analysis as it showed that an assumed value of 75 MPa would yield a very similar behavior for crack propagation as was seen in the experiment.
The effect of the magnitude of the compressive residual stresses on $K_{\text{eff}}$, could easily be seen in Figure 6 (Dexter and Mahmoud, 2004). The gap between $K_{\text{app}}$ and $K_{\text{eff}}$ in the case of compressive stresses of 100 MPa is higher than the case of 50 MPa. Obviously, as the magnitude of the compressive residual stresses goes to zero, $K_{\text{eff}}$ would be equal to $K_{\text{app}}$. This comparison shows how sensitive the analysis is to the magnitude of the residual stresses. For stress ranges near the threshold, overestimation of compressive residual stress can significantly overestimate the remaining number of fatigue cycles. Therefore, it is worthwhile to note that any assumption for compressive residual stress field be a lower bound in real world applications.

Figure 6 - $\Delta K_{\text{app}}$ & $\Delta K_{\text{eff}}$ for compressive residual stresses of 100 MPa (left) and 50 Mpa (right)

5.0 Experimental Studies

Dexter and Pilarski (2000) conducted a series of experiments on stiffened box-sections in four-point bending as shown in Figure 7. Testing in four point bending created a region of constant moment across the stiffened box section. It is however important to note that bending gradient did exist through the depth of the box due to shear lag in the specimen. The specimen was 3050 mm in length with a width of 1370 mm and a plate thickness of 12.7-mm. A typical specimen cross section is shown in Figure 7. Continuous angle or tee stiffeners were attached to the bottom plate of the box section with double fillet welds (Case 1). The stiffeners were welded to the bottom plate before the top plate and side webs were added to complete the full configuration of the specimen. Two specimens had a 25 mm radius semicircular drain hole along the crack line (Case 2). Another specimen had a raised drain hole, 37 mm long and 19 mm in width, with an edge distance of 13 mm between the plate surface and the edge of the hole (Case 3). Several specim
incorporated a butt weld typical of an erection butt weld used to join ship modules (Case 4).

![Four-point bending test fixture](image1)

*Figure 7 - Four-point bending test fixture (left) and typical cross section of stiffened box-section (right)*

The crack was started in each specimen from a saw cut in the middle of the bottom plate between the interior stiffeners. The predicted crack length vs. number of cycles for case 1 through case 4 is shown in Figure 8. As the figure shows, stable crack propagation was observed in all tested specimens. Crack initiation and growth in the specimen with typical butt weld used to join ship modules (case 4) was similar if not slightly higher to than the propagation rate in a tested center crack tension panel with no stiffeners (not shown in the figure), and significantly higher than that of the other specimens. Such ease of crack initiation and propagation demonstrates the fatigue sensitivity of this type of detail in ship structure. While unstable crack growth was never observed, the beneficial effects of any internal compressive stress due to the stiffener fillet welds were negated by the butt weld. Furthermore, the stiffeners themselves provided no retardation effects on the crack growth. The number of cycles it took for a crack to propagate between stiffeners is approximately twice as much as the specimen with transverse butt weld and baseline case of a CCT specimen with no stiffeners. This result shows that redundancy and residual stresses were beneficial in slowing the crack propagation rate (decreased by a factor of two).
To quantify the effect of bending gradient through the depth of the panel and the effect of redundancy on crack growth rate, five single non-redundant stiffened plates were tested in direct tension as shown in Figure 9 (Dexter and Mahmoud, 2004). Two different types of specimens were fabricated. The specimens differed in their stiffener type, either angle or bulb-tee stiffeners, and the intensity of the heat input used for attaching the stiffeners to the panels. One plate, S4, which was 1626 mm wide and 13 mm thick, was fabricated with angle stiffeners, which were attached to the panel using medium heat input in the welding process. The angle stiffeners were 101 mm x 76 mm x 8 mm, with a spacing of 381 mm. Four specimens were fabricated with bulb-tee stiffeners (HP 160 x 9). Two of which, S1 and S2, had a plate thickness of 13 mm and a stiffener spacing of 381 mm. Medium heat input was used for attaching the stiffeners in specimen S1, while high heat input was used in specimen S2. Specimen S5 was similar to S1 except the plate thickness was 9 mm. Medium heat input used in welding the stiffeners. Specimen S3 had a 13 mm thick plate and stiffener spacing of 305 mm. Medium heat input was used in the welding process. Figure 9 shows typical specimen dimensions and the stiffeners’ spacing.
Similar to the method followed previously by Dexter and Pilarski (2000), the crack was started in each specimen from a saw cut in the middle of the plate between the interior stiffeners. Furthermore, a tack weld was deposited over the tip of the initial cut to accelerate the propagation of the crack. The predicted crack growth versus number of cycles is shown in Figure 10 and Figure 11. Note that the predicted crack growth rate for specimen S3 is shown separately in Figure 11 since specimen S3 had different stiffener spacing than the remaining specimens. As in the previous experimental projects conducted on stiffened box sections, the experiments confirmed that stable crack propagation behavior in single non-redundant stiffened panels can be relied upon and can be conservatively predicted. The behavior in these experiments was not significantly different from those performed in the previous project in bending; indicating that the lack of bending stress gradient and redundancy did not alter this basic conclusion.
Figure 10 - Crack extension vs. cycles for specimens S1, S2, S4, and S5 (left) and physical crack in S1 (right)

Figure 11 - Crack extension vs. cycles for specimens S3

5.0 Conclusion

1. Unless the crack is propagating along a butt weld, welded stiffeners substantially reduce the crack propagation rate relative to a plate with no stiffeners. The cycles to propagate one stiffener spacing may increase by a factor of 2 to 4 above what would be predicted for a center-cracked plate.

2. Transverse crack propagation in stiffened box-sections or stiffened single panels with no transverse butt welds can be reasonably predicted by including the }
splitting forces from the severed stiffeners and the effect of compressive residual stresses between the stiffeners.

3. Numerical and analytical models were developed that could predict the stable crack propagation with reasonable accuracy based on solutions for the stress intensity factor range and the Paris Law for crack propagation rate.

4. Crack propagation rate observed in highly redundant box sections loaded in bending is similar to the propagation rate in a single non-redundant panels suggesting that redundancy and stress gradient in the box section tested previously had little effect on the propagation rate.

5. Measured residual stress showed high variability in magnitude with a consistent pattern of tensile residual stress adjacent to the stiffeners and compressive residual stress between the stiffeners. The pattern is consistent with past studies of residual stress in welded stiffened panels.

References


